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A supply chain network design considering transportation cost discounts

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A B S T R A C T

This study addresses an integrated facility location and inventory allocation problem considering transportation cost discounts. Specifically, this article considers two types of transportation discounts simultaneously: quantity discounts for inbound transportation cost and distance discounts for outbound transportation cost. This study uses an approximation procedure to simplify DC distance calculation details, and develops an algorithm to solve the aforementioned supply chain management (SCM) problems using nonlinear optimization techniques. Numerical studies illustrate the solution procedures and the effects of the model parameters on the SCM decisions and total costs. Results of this study serve as a reference for business managers and administrators.

1. Introduction

The value of inventory is approximately 14% of gross domestic product (GDP) in the United States, while annual transportation and warehousing expenses average approximately nine percent of GDP (Wilson, 2005). Retail companies in the US spend approximately $14 billion per year on inventory interest, insurance, taxes, depreciation, obsolescence, and warehousing. Their logistics activities account for 15–20% of the total cost of finished goods (Menlo, 2007). With such a huge logistics investment, it is important to make sound decisions for facility locations and inventory allocation in a supply chain (SC). The design and management of SC network in today's competitive business environment is one of the most important and difficult problems that managers face.

In today's business environment most retail companies have complex distribution networks with several national and regional distribution centers (DCs). For example, Target, Inc. has three import warehouses, 22 regional distribution centers, and 1300 retail stores. Frito-Lay, Inc. operates its distribution network with 42 plants, one national DC, and 325 regional DCs (Erlebacher and Meller, 2000). When goods arrive at US seaports, they must be consolidated by regions at national (import) DCs. From these national distribution centers (NDCs), goods are shipped to regional distribution centers (RDCs), from which they are delivered to retail stores. There exists a substantial cost in transportation goods from NDCs to RDCs and from RDCs to retailers. In practice, discount for larger quantity of freight or longer distance of shipment may be applicable to transportation economies of scale (Shinn et al., 1996). Since logistics cost plays a key factor in SC design and management decisions, incorporating transportation cost discounts into SC network design problem is necessary. The current paper is the first SC network design study to consider transportation cost discounts.

Since the SC network is a large-scale complex system, detailed modeling and optimization are difficult (e.g., see several examples of detailed discrete modeling in Section 2). Since our focus in this study is the strategic decision making, this article

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presents a two-phase approximation technique to solve the SC network design problem. This approximation leads to a substantial reduction in the amount of data characterizing the SC system. The resulted simplified system is then much easier for optimization and comparison studies to explore managerial insights. Specifically, the proposed solution formulates transportation traveling distance and cost with continuous functions of the sizes of influential area covered by DCs. Since the SC network design involves multiple parameters and the objective function could be nonlinear with bounded constraints, this study provides heuristic algorithms to solve this optimization problem. The following provide details to further support the approximation approach.

While discrete models (see examples in Section 2) can provide managers with optimal solutions, their data and computational requirements increase tremendously as they become more realistic. Moreover, data reliability and model accuracy are of concerns in practice, especially in dynamically changing business environment. The key idea of the continuous approximation (CA) approach is to define decision variables using continuous functions for reducing SCM problem complexity. Although the CA approach does not determine the exact location of the distribution centers, it defines a service area for each distribution center in terms of circular influence areas. Studies by Newell (1973) (and Dasci and Verter, 2001) showed that influence area with central distribution nodes is a near optimal solution. See Section 2 for more examples of CA approaches in solving SCM problems.

The goal of this study is to provide logistics network planners with a high-level solution for the integrated facility location and inventory allocation problem under quantity and distance discounts for transportation costs. Chen (2010) and Chen and Chang (2010) emphasized the importance of integrated decision-making. Specifically, this study intends to determine the following SC network design decisions: (1) which RDC locations should be open, (2) which retail store should be served from which RDC, and (3) how much inventory should be held at the NDCs and the RDCs? We consider both situations when order quantity is the same across all RDCs and when order quantity is different at RDCs in different regions. Two heuristic algorithms are provided to solve the problems. Numerical study illustrates the solution procedures and impacts of the relevant model parameters on SC design decisions and profits.

The remainder of this study is organized as follows. Section 2 reviews key literature relevant to the studies. Section 3 describes the problem including assumptions and notations. Section 4 formulates the model and develops a heuristic algorithm for solving the problem. Section 5 presents numerical examples and analyses to illustrate the solution procedure and impact of changing system parameters. Section 6 extends the procedure to solve more realistic problems. Finally, Section 7 concludes the study.

2. Literature review

2.1. Supply chain network design

In recent years, many retail companies have explored better ways for designing and managing their SC for achieving cost savings. There are several publications in the area of integrated facility location and inventory decisions. Shen (2007) has made a complete review of the supply chain design literature and of current practices. Daskin's (1995) fixed-charge facility location model uses the linear inventory cost function to determine DC locations achieving the least cost. Nozick and Turnquist (1998) approximated the safety stock cost at each DC using a linear regression function of the number of DCs. They then used this function to estimate the inventory cost function. Their model stocks inventory at the DC and replenishes it using a one-for-one policy. Nozick and Turnquist (2001) extended their previous model by adding service responsiveness and uncertainty to DC delivery time. They defined service responsiveness in terms of stock-outs and time-based delivery. Stock-outs are incorporated in the safety stock function while the time-based delivery constraint is modeled explicitly as coverage distance.

Daskin et al. (2002) and Shen et al. (2003) proposed a set-covering model to consider location and allocation policies for a DC-retailer network with risk pooling. They successfully showed that this problem can be solved efficiently when the DC demand is deterministic or Poisson distributed. Shu et al. (2005) extended this model to consider arbitrary demand distributions. They presented computational results for several instances, with sizes ranging from 40 to 500 retailers. Shen (2005) considered a location-allocation problem for a multi-commodity supply chain. Shen and Daskin (2007) extended the nonlinear integrated location-inventory model to incorporate a measure of customer service quality. Shen and Qi (2007) removed the assumption in Shen et al. (2003). They modeled the shipment from a DC to its customers using a vehicle routing model instead of the linear direct shipping model, and proposed a Lagrangian relaxation based solution algorithm. Javid and Azad (2010) established a heuristic method based on a hybridization of Tabu Search and Simulated Annealing to solve the location, routing and inventory problem.

For location-allocation and inventory policies, Teo and Shu (2004) proposed a set-covering model to design a two-echelon warehouse-retailer network under deterministic retailers demands. Their problem was to determine the optimal warehouses locations, allocate retailers to warehouses, and make inventory decisions for warehouses and retailers. Their objective was to minimize the total two-echelon inventory, transportation, and facility location cost. They provided computational results for problems involving 20 warehouses and 100 retailers. Romeijn et al. (2007) extended the problem to consider an additional cost term that may represent costs related to safety stocks or capacity considerations. They studied the structure of the pricing subproblem and developed an algorithm to solve it, providing computational results for problems with 10 or 20 DCs and 10–70 retailers. Snyder et al. (2007) presented a stochastic location model with risk pooling and developed a Lagrangian-relaxation-based exact algorithm to solve it. Their goal was to determine optimal DC locations, assign retailers to DCs, and set inventory levels at DCs to minimize the total expected cost. Naseraldin and Herer (2008) believed both retail outlets and customers are located on a finite homogenous line segment. They determined the optimal values of the number of retail outlets, the location of each retail outlet, and the replenishment inventory levels at each retail outlet. Park et al. (2010) considered a single-sourcing network design problem for a three-level supply chain where risk-pooling strategy and DC-to-supplier dependent lead times are considered. The focus of our study is to simultaneously determine RDC locations, retail store allocation, and inventory level at the NDCs and the RDCs. The study is more comprehensive and covers transportation cost discount issues, which are usually not considered in supply chain network designs.

2.2. Transportation cost discount

Transportation is a significant component of supply chain operations. Considering transportation costs in inventory replenishment decisions can reduce the total SC cost (Toptal, 2009). In many practical situations, discounts for larger quantities of freight may be applicable to transportation economies of scale, in terms of the number of unit loads delivered (Shinn et al., 1996). Lee (1986) considered discounted per-truck costs for larger replenishment quantities. Shinn et al. (1996) determined pricing and ordering decisions under discounted freight costs and delay in payments. Sheen and Tsao (2007) discussed channel coordination issue under trade credit and freight cost discounts. They assumed that the transportation cost includes quantity discounts due to economies of scale. Glickman and White (2008) addressed optimal vendor selection problem in a multiproduct supply chain with truckload discount. Our study considers quantity discounts for transportation cost between NDCs and RDCs.

Distance is another important dimension of transportation charge. Transportation cost for service vary with the distance over which the freight must be transferred (Ballou, 2004)). This is reasonable because the amount of fuel used depends on distance, and the amount of labor is a function of distance. Thus, the longer transport distance the products will be transferred, the lower the unit distance transportation cost will be. Our study assumes that the transportation cost consists of a fixed cost and an additional variable cost paid per unit distance. This study considers the distance discounts for transportation cost between RDCs and retailers. This is the first study to consider two different transportation discounts simultaneously in the supply chain network design problem.

2.3. Continuous approximation (CA) approach

Continuous approximation models, which use continuous functions to represent distribution of retailer location and demand, have been developed to provide insights into complicated mathematical programming models (Shen, 2007). It is also widely recognized that continuous models should supplement mathematical programming models (Hall, 1986). Geoffrion (1976) studied a continuous model for warehouse location in which a warehouse serves demand that is distributed uniformly over a plane. Erlenkotter (1989) used a General Optimal Market Area model to determine optimal area served by a single production point when demand is assumed to be distributed uniformly. Daganzo (1996) presented CA techniques for network designing problem, and focused on vehicle dispatch scheduling. Langevin et al. (1996) reviewed CA approaches and developed a method for solving freight distribution problems. They showed that combination of CA models and optimization methods can be a powerful tool for SCM studies. Erlebacher and Meller (2000) used a CA approach to formulate a non-linear integer location/inventory model. Pujari et al. (2008) utilized a CA procedure to determine optimal number and size of shipments while considering issues of location, production, inventory, and transportation. Murat et al. (2010) provided a CA framework for solving location-allocation problems with dense demand. Murat et al. (2011) formulated the two-facility location-allocation problem as a multi-dimensional boundary value problem and developed a multi-dimensional shooting algorithm to solve it. Their methodology is suitable for problems where allocation decisions can be reasonably approximated by polygons. These examples show that CA approaches are becoming common today in solving SCM problems, especially for large size data. In our study, the proposed solution defines the input data in terms of continuous functions and is capable of formulating these functions for a data set of any size.

3. Problem description

The network studied in this study is a three-echelon supply chain with an outside supplier at level three selling goods to NDCs. The NDCs are located at level two, and help consolidate shipments arriving from overseas manufacturers and deliver them to the RDCs. The RDCs are located at level one, and help consolidate shipments and pool risk. The retail stores at level zero meet the demands from end customers. Goods flow from higher-level facilities to the lower-level facilities until they reach level zero (see Fig. 1).

The mathematical model in this study is based on the following assumptions:

1. The location of the NDC is known and fixed.

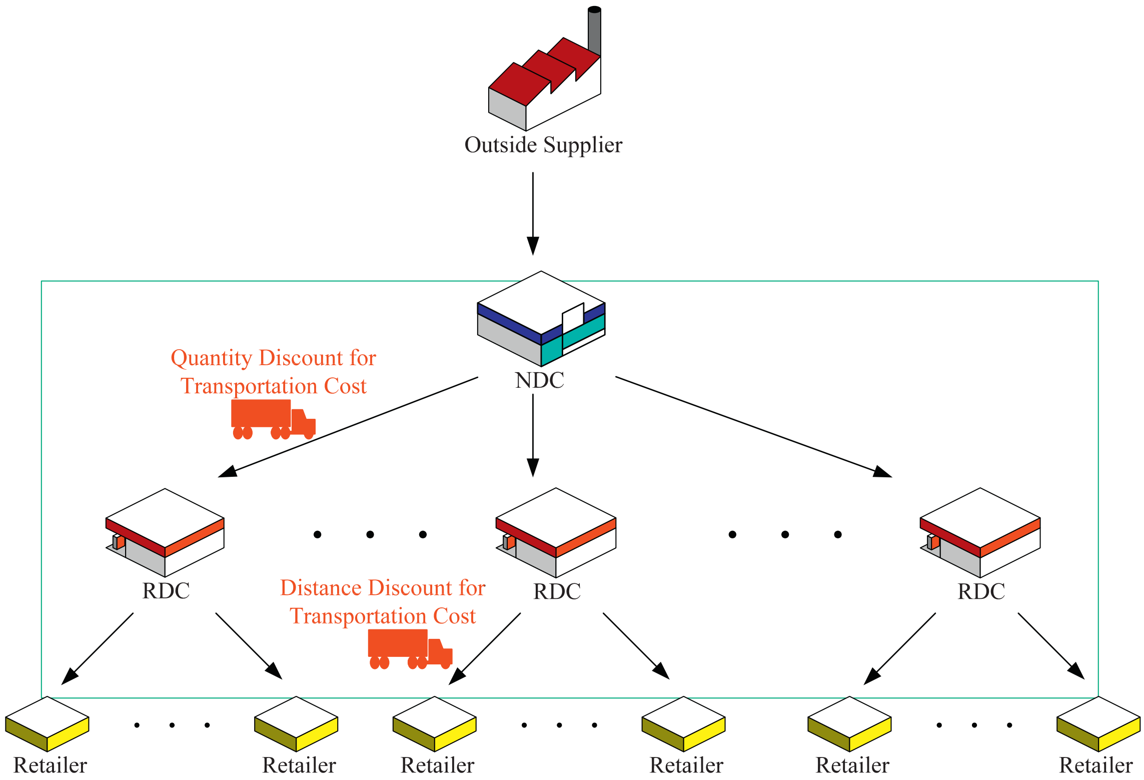

Fig. 1. Multi-level supply chain network.

2. Demand per unit time for retail store in cluster C_i is an independent and identically distributed Poisson process with rate λ_i.
3. Each RDC's influence area is close to circular. Service regions have somewhat irregular shapes as opposed to circles, hexagons, or squares in the economics literature. This irregular service area is shown to have little effect on the optimal solution (Dasci and Verter, 2001). Moreover, each RDC is located in the center of the influence area.
4. The constraints from capacity limitations at the NDCs and the RDCs are ignored.
5. It is a common practice in multi-echelon inventory studies to assume that the order quantity Q_r is the same across all RDCs (Deuermeyer and Schwarz, 1981 and Ganeshan, 1999).
6. Replenishments occur in very short time for the RDC-Retailer echelon, i.e. lead time can be ignored for the RDC-Retailer echelon.
7. The expected lead time for the NDC-RDC echelon is μ.
8. Each retailer is assigned to a particular RDC and served only by that RDC.
9. The freight carrier offers quantity discounts for transportation cost between NDCs and RDCs due to economies of scale.
10. For transportation cost between RDCs and retailers, the transportation cost contains a fixed cost and an additional variable cost paid per unit distance.

This study uses the following notations.

F_r	facility cost of opening each RDC r
A_{r_i}	influence area for each RDC r in cluster C_i, where $i = 1,2,\ldots,N$
δ_i	store density in cluster C_i
λ_i	demand rate for retail store in cluster C_i
ξ	length of the planning horizon
Q_n	ordering quantity for NDC
Q_r	ordering quantity for RDC r in each cluster
T_j	inbound transportation cost for Q_r in cluster C_i, $B_{j-1} < Q_r \leqslant B_j$, where $T_{j-1} < T_j$ and $T_{j-1}/B_{j-1} > T_j/B_j$, $j = 1,2,\ldots,n$
B_j	jth inbound transportation cost break quantity, $j = 1,2,\ldots,n$, where $B_0 < B_1 < \cdots < B_n < B_{n+1}$ with $B_0 = 0$ and $B_{n+1} = \infty$
C_f	fixed transportation cost per item

C_v variable transportation cost per mile per item
f_r constant that depends on the distance metric and shape of the RDC r service region
R_n ordering cost for NDC
R_r ordering cost for each RDC r
h_n inventory holding cost for NDC
h_r inventory holding cost for each RDC r
μ expected lead time for the NDC-RDC echelon
C_i cluster i in the given NDC partition

First, consider a given NDC partition and suppose that $(C_1, C_2, \ldots, C_N)$ are the clusters within the given NDC partition. Let A_{r_i} be the size of the influence area for each RDC in cluster C_i. This model calculates the components of the total network cost as follows.

(1) The total facility cost is given by multiplying the facility cost of opening each RDC with the number of RDCs, namely, $F_r \frac{C_i}{A_{r_i}}$.

(2) Consider two components for the transportation cost: inbound and outbound costs. The inbound cost is the cost of sending shipments from the NDC to the RDC. The outbound cost is the cost of shipping goods from the RDC to the retailers located within its influence area.

 (2.1) The total inbound transportation cost is $\sum_{i=1}^{N}\left(T_j \frac{\xi\lambda_i\delta_i C_i}{Q_r}\right)$, where $T_j = a \times j \times [1 - b(j-1)]$ is the inbound transportation cost for quantity Q_r, a and b are positive constants and $j = 1, 2, \ldots, n$. When the ordering quantity Q_r is within $B_{j-1} < Q_r \leqslant B_j$, the inbound transportation cost for Q_r is T_j. The form of T_j is refer to Lee (1986), Shinn et al. (1996) and Sheen and Tsao (2007).

 (2.2) Assuming "close to circular" service regions with the facility at the center, the average outbound distance traveled by each item is $f_r\sqrt{A_r(x)}$ (Dasci and Verter, 2001). For each item, the outbound transportation cost is $C_f + C_v f_r\sqrt{A_{r_i}}$ which includes a fixed transportation cost C_f and a variable transportation cost $C_v f_r\sqrt{A_{r_i}}$ based on the outbound distance traveled. Thus, the outbound transportation cost satisfies that the longer transport distance the products will be transferred, the lower the unit distance transportation cost will be. The outbound transportation cost for all items in cluster C_i is $(C_f + C_v f_r\sqrt{A_{r_i}})(\xi\lambda_i\delta_i C_i)$. Therefore, the total outbound transportation cost for the given NDC partition is $\sum_{i=1}^{N}\left[(C_f + C_v f_r\sqrt{A_{r_i}})(\xi\lambda_i\delta_i C_i)\right]$.

(3) The total NDC ordering cost is $\sum_{i=1}^{N}\left(R_n \frac{\xi\lambda_i\delta_i C_i}{Q_n}\right)$.

(4) The total RDC ordering cost is $\sum_{i=1}^{N}\left(R_r \frac{\xi\lambda_i\delta_i C_i}{Q_r}\right)$.

(5) From Deuermeyer and Schwarz (1981) and Mangotra et al. (2009), the total NDC inventory holding cost includes regular stock cot and safety stock cost: $h_n\left(\frac{Q_n}{2} + Z_\alpha\sqrt{\sum_{i=1}^{N}\mu\lambda_i\delta_i C_i}\right)$, where α is the service level at the NDC.

(6) The total RDC inventory holding cost is $\sum_{i=1}^{N}\left(\frac{C_i}{A_{r_i}}\frac{h_r Q_r}{2}\right)$, where $\frac{C_i}{A_{r_i}}$ is the number of RDCs in cluster C_i, where $i = 1, 2, \ldots, N$. The quantity $\frac{h_r Q_r}{2}$ is the inventory holding cost for a RDC. Therefore, the total RDC inventory holding cost for all clusters is $\sum_{i=1}^{N}\left(\frac{C_i}{A_{r_i}}\frac{h_r Q_r}{2}\right)$.

Therefore, the total network cost is

$$TNC(A_{r_i}, Q_n, Q_r) = \sum_{i=1}^{N}\left(F_r\frac{C_i}{A_{r_i}}\right) + \sum_{i=1}^{N}\left(T_j\frac{\xi\lambda_i\delta_i C_i}{Q_r}\right) + \sum_{i=1}^{N}\left[(C_f + C_v f_r\sqrt{A_{r_i}})(\xi\lambda_i\delta_i C_i)\right] + \sum_{i=1}^{N}\left(R_n\frac{\xi\lambda_i\delta_i C_i}{Q_n}\right) + \sum_{i=1}^{N}\left(R_r\frac{\xi\lambda_i\delta_i C_i}{Q_r}\right) + h_n\left(\frac{Q_n}{2} + Z_\alpha\sqrt{\sum_{i=1}^{N}\mu\lambda_i\delta_i C_i}\right) + \sum_{i=1}^{N}\left(h_r\frac{C_i}{A_{r_i}}\frac{Q_r}{2}\right). \quad (1)$$

4. Solution methodology

This study uses a two-phase approximation technique (Mangotra et al., 2009) to solve the supply network design problem. The main idea for the two-phase approximation method is to divide the network into smaller regions over which the discrete variable can be modeled using the slow varying functions. Phase-I approximation uses the Grid Cover-Couple approach to partition the service region into sub-regions. A mesh of equal sized squares is designed to cover the given NDC partition. The geometry of the square-mesh needs to satisfy that the demand is slow varying within each grid square. Within the given NDC partition, there are grids and each grid has a density associated with it. The grids with similar densities can be clustered together to form areas over which the store density function is slow varying. Using the method the given NDC partition is covered with clusters ($C_1, C_2, \ldots, C_N$). Clusters ($C_1, C_2, \ldots, C_N$) exist within the given NDC partition such that the store density is nearly constant over each cluster.

Phase-II approximation uses the continuous approximation technique to model the facility location and inventory allocation problem over each cluster within the NDC partition. Section 3 models the total network costs. Using the size of the optimal influence area along with the information on the area for each cluster, the total number of RDCs in each cluster can be calculated. The total number of RDCs in the NDC partition is obtained by summing over the number of RDCs in each cluster. For more information about Phase-I and Phase-II approximations, please see Mangotra et al. (2009).

The problem analyzed here is to determine the optimal influence area for each RDC A_{r_i}, ordering quantity for each RDC Q_r and ordering quantity for NDC Q_n to minimize total network cost $TNC(A_{r_i}, Q_n, Q_r)$. Since Q_r is affected by quantity discounts for transportation cost, first deal with the decisions A_{r_i} and Q_n under a given Q_{r_i}. For a given Q_{r_i}, we have

$$\frac{\partial^2 TNC(A_{r_i}, Q_n|Q_r)}{\partial A_{r_i}^2} = \sum_{i=1}^{N}\left(\frac{2F_rC_i}{A_{r_i}^3}\right) + \sum_{i=1}^{N}\left(\frac{-C_v f_r \xi \lambda_i \delta_i C_i}{4A_{r_i}^{3/2}}\right) + \sum_{i=1}^{N}\left(\frac{h_r C_i Q_r}{A_{r_i}^3}\right), i = 1, 2, \ldots, N,$$

$$\frac{\partial^2 TNC(A_{r_i}, Q_n|Q_r)}{\partial Q_n^2} = 2R_n \frac{\sum_{i=1}^{N}(\xi\lambda_i\delta_i C_i)}{Q_n^3} > 0,$$

$$\frac{\partial^2 TNC(A_{r_i}, Q_n|Q_r)}{\partial A_{r_i}\partial A_{r_j}} = 0, j = 1, 2, \ldots, N, \quad \text{but } i \neq qj,$$

$$\frac{\partial^2 TNC(A_{r_i}, Q_n|Q_r)}{\partial A_{r_i}\partial Q_n} = 0.$$

The threshold of F_r is

$$\left\{\sum_{i=1}^{N}\left(\frac{C_v f_r \xi \lambda_i \delta_i C_i}{4A_{r_i}^{3/2}}\right) - \sum_{i=1}^{N}\left(\frac{h_r C_i Q_r}{A_{r_i}^3}\right)\right\} \Big/ \sum_{i=1}^{N}\left(\frac{2C_i}{A_{r_i}^3}\right).$$

This means

$$\frac{\partial^2 TNC(A_{r_i}, Q_n|Q_r)}{\partial A_{r_i}^2} > 0$$

when

$$F_r > \left\{\sum_{i=1}^{N}\left(\frac{C_v f_r \xi \lambda_i \delta_i C_i}{4A_{r_i}^{3/2}}\right) - \sum_{i=1}^{N}\left(\frac{h_r C_i Q_r}{A_{r_i}^3}\right)\right\} \Big/ \sum_{i=1}^{N}\left(\frac{2C_i}{A_{r_i}^3}\right).$$

Since the facility opening cost F_r is large, $\frac{\partial^2 TNC(A_{r_i}, Q_n|Q_r)}{\partial A_{r_i}^2}$ >0 is satisfied in the general case. The Hessian matrix is then

$$H_i = \begin{bmatrix} \frac{\partial^2 TNC}{\partial A_{r_1}^2} & 0 & \cdots & 0 & 0 \\ 0 & \frac{\partial^2 TNC}{\partial A_{r_2}^2} & \cdots & \cdot & 0 \\ \vdots & & \ddots & \vdots & \vdots \\ \vdots & & & 0 & \vdots \\ 0 & \cdot & \cdots\ 0 & \frac{\partial^2 TNC}{\partial A_{r_N}^2} & 0 \\ 0 & 0 & \cdots & 0 & \frac{\partial^2 TNC}{\partial Q_n^2} \end{bmatrix}, \quad i = 1, 2, \ldots, N+1.$$

Since $\frac{\partial^2 TNC}{\partial A_{r_i}^2} > 0$ and $\frac{\partial^2 TNC}{\partial Q_n^2} > 0$, we know that $|H_i| > 0$ for $i = 1, 2, \ldots, N + 1$. From the minimum theorem in Winston (2004), we know that $TNC(A_{r_i}, Q_n|Q_r)$ is a convex function of A_{r_i} and Q_n. This means that the optimal A_{r_i} and Q_n can be obtained by solving $\frac{\partial TNC(A_{r_i}, Q_n|Q_r)}{\partial A_{r_i}} = 0$ and $\frac{\partial TNC(A_{r_i}, Q_n|Q_r)}{\partial Q_n} = 0$:

$$A_{r_i}(Q_r) = \left(\frac{2F_r + h_r Q_r}{C_v f_r \xi \lambda_i \delta_i}\right)^{2/3}, \tag{2}$$

$$Q_n = \sqrt{\frac{2R_n \sum_{i=1}^{N} \xi \lambda_i \delta_i C_i}{h_n}}. \tag{3}$$

Eqs. (2) and (3) lead to Property 1.

Property 1.

(a) *The influence area for each RDC A_{r_i} increases as the ordering quantity for RDC Q_r or the facility cost of opening each RDC F_r increase.*
(b) *The ordering quantity for NDC Q_n increases as the inventory holding cost for NDC h_n decreases or the ordering cost for NDC R_n increases.*
(c) *The influence area for each RDC A_{r_i} decreases as variable transportation cost C_v increases. However, A_{r_i} will not change as C_f changes.*

Substituting Eqs. (2) and (3) into Eq. (1) yields

$$TNC(Q_r) = \sum_{i=1}^{N}\left[F_rC_i\left(\frac{C_vf_r\xi\lambda_i\delta_i}{2F_r+h_rQ_r}\right)^{2/3}\right] + \sum_{i=1}^{N}\left\{\left[C_f + C_vf_r\left(\frac{2F_r+h_rQ_r}{C_vf_r\xi\lambda_i\delta_i}\right)^{1/3}\right](\xi\lambda_i\delta_iC_i)\right\} + \sum_{i=1}^{N}\left[\frac{h_rC_iQ_r}{2}\left(\frac{C_vf_r\xi\lambda_i\delta_i}{2F_r+h_rQ_r}\right)^{2/3}\right] + \sqrt{2h_nR_n\sum_{i=1}^{N}(\xi\lambda_i\delta_iC_i)} + h_nZ_\alpha\sqrt{\sum_{i=1}^{N}\mu\lambda_i\delta_iC_i} + \sum_{i=1}^{N}\left(R_r\frac{\xi\lambda_i\delta_iC_i}{Q_r}\right) + \sum_{i=1}^{N}\left(T_j\frac{\xi\lambda_i\delta_iC_i}{Q_r}\right). \tag{4}$$

For a specific T_j with respect to $B_j < Q_r \leqslant B_{j+1}$, determine the optimal Q_r^*, which minimizes $TNC(Q_r)$. If the optimal Q_r^* satisfies $B_j < Q_r \leqslant B_{j+1}$, it is the valid optimal Q_r^* with respect to $B_j < Q_r \leqslant B_{j+1}$. If this optimal Q_r^* does not satisfy $B_j < Q_r \leqslant B_{j+1}$, it is an invalid Q_r^* with respect to $B_j < Q_r \leqslant B_{j+1}$. Consider the two following cases.

4.1. Case A: a valid optimal Q_r^* is found

Assuming that a valid optimal Q_r^* falls in the range of $B_{j^*-1} < Q_r \leqslant B_{j^*}$, we have the following lemma.

Lemma 1. *If the valid optimal Q_r^* with respect to $B_{j^*-1} < Q_r \leqslant B_{j^*}$ is found, the optimal Q_r^{**} which minimizes TNC(Q_r) occurs at this valid optimal Q_r^* or at B_j with $j < j^*$.*

Proof. Because $T_j < T_{j+1}$, the minimal *TNC* associated with T_j is less than the minimal *TNC* associated with T_{j+1}. Therefore, if the valid optimal Q_r^* with respect to $B_{j^*-1} < Q_r \leqslant B_{j^*}$ is found, we do not need to search for corresponding Q_r^* in ranges beyond B_{j^*}. When the valid optimal Q_r^* with respect to $B_{j^*-1} < Q_r \leqslant B_{j^*}$ is found, each corresponding $Q_r^* > B_j$ holds for any j: $1 < j \leqslant j^* - 1$. In other words, Eq. (4) yields an invalid optimal Q_r^* with respect to $B_{j^*-1} < Q_r \leqslant B_{j^*}$ for any j, $1 < j \leqslant j^* - 1$. Assume that Q_r^* is the point for minimizing $TNC(Q_r)$. Fig. 2 indicates that $TNC(Q_r)$ is a convex–concave function of Q_r, where $TNC(Q_r)$ is convex for $Q_r < Q_r^I$ and concave for $Q_r > Q_r^I$. $TNC(Q_r)$ is a decreasing function of Q_r when $0 < Q_r < Q_r^*$. Because $Q_r^* > B_j$ holds for any j, $1 < j \leqslant j^* - 1$, the minimum value of $TNC(Q_r)$ occurs at B_j for any j, $1 < j \leqslant j^* - 1$. Therefore, it is only necessary to consider Q_r^* and B_j with all $j < j^*$ as candidates for the optimal Q_r^{**}. □

Lemma 1 states that the optimal Q_r^{**} may occur at the valid optimal Q_r^* or at B_j with $j < j^*$. If the order quantity Q_r is B_j, the optimal influence area for RDC r is $A_{r_i}^*(B_j)$ (in Eq. (2) Q_r is replaced by B_j). Therefore, substituting B_j, Q_n and $A_{r_i}(B_j)$ into Eq. (1) leads to

$$TNC(N_j) = \sum_{i=1}^{N}\left[F_rC_i\left(\frac{C_vf_r\xi\lambda_i\delta_i}{2F_r+h_rB_j}\right)^{2/3}\right] + \sum_{i=1}^{N}\left\{\left[C_f + C_vf_r\left(\frac{2F_r+h_rB_j}{C_vf_r\xi\lambda_i\delta_i}\right)^{1/3}\right](\xi\lambda_i\delta_iC_i)\right\} + \sum_{i=1}^{N}\left[\frac{h_rC_iB_j}{2}\left(\frac{C_vf_r\xi\lambda_i\delta_i}{2F_r+h_rB_j}\right)^{2/3}\right] + \sqrt{2h_nR_n\sum_{i=1}^{N}(\xi\lambda_i\delta_iC_i)} + h_nZ_\alpha\sqrt{\sum_{i=1}^{N}\mu\lambda_i\delta_iC_i} + \sum_{i=1}^{N}\left(R_r\frac{\xi\lambda_i\delta_iC_i}{B_j}\right) + \sum_{i=1}^{N}\left(T_{i,j}\frac{\xi\lambda_i\delta_iC_i}{B_j}\right). \tag{5}$$

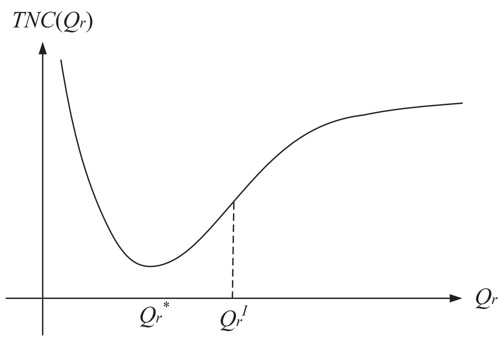

Fig. 2. Shape of $TNC(Q_r)$.

It is then possible to search for an optimal B_j to minimize $TNC(B_j)$, and, in turn, calculate the optimal values of $A_{r_i}(B_j),\ Q_n$ and $TNC(B_j)$.

4.2. Case B: a valid optimal Q_r^* is not found

If a valid optimal Q_r^* is not found, the Q_r that minimizes $TNC(Q_r)$ with respect to each range of $B_{j-1} < Q_r \leqslant B_j$ is B_j. Therefore, the optimal Q_r^{**} that minimizes $TNC(Q_r)$ can be found by comparing all values of $TNC(B_j)$ with $j \leqslant n$. The way to determine decisions and cost when the order quantity is $Q_r = B_j$ is the same as in Case A.

Based on the formulation for both of these cases, this study uses the following heuristic algorithm to obtain the optimal ordering quantity Q_r^{**}.

Algorithm 1.

Step 1. Beginning with the freight cost $T_{j=1}$, search for the order quantity Q_r that minimizes Eq. (5) until a valid optimal Q_r^* is found (i.e. Q_r^* must fall within the corresponding break quantity range, $B_{j-1} < Q_r \leqslant B_j$) or $j = n$.
Step 2. If a valid optimal Q_r^* is found, let the corresponding break quantity range be $B_{j^*-1} < Q \leqslant B_{j^*}$ and go to Step 3.1; otherwise go to Step 3.2.
Step 3. Select the optimal order quantity.
  3.1: Compare the total network cost $TNC(Q_r^*)$ and $TNC(B_j)$ with all $j < j^*$. Select the value (Q_r^* or B_j) to minimize these costs; then stop.
  3.2: Compare the annual profit $TNC(B_j)$ for all $j \leqslant n$. Select the value B_j that minimizes the total network cost; then stop.

5. Numerical study

This section presents numerical study to illustrate the proposed solution approach and provide quantitative insights. The goals of the numerical study in this study are as follows:

1. To illustrate the procedures of the solution approach;
2. To discuss the impacts of the related parameters on decisions and cost.

5.1. Numerical example

To illustrate the algorithm described above, consider the parameters of a product in a distribution company: $F_r = 100{,}000$; $C_1 = 10{,}000$; $C_2 = 8000$; $C_3 = 6000$; $h_r = 1$; $h_n = 1$; $R_r = 30$; $R_n = 30$; $C_f = 10$; $C_v = 10$; $\lambda_1 = 11$; $\lambda_2 = 10$; $\lambda_3 = 9$; $\xi = 12$; $\delta_1 = 0.06$; $\delta_2 = 0.05$; $\delta_3 = 0.04$; $f_r = 0.01$; $Z_{0.9} = 1.645$, $\mu = 0.0083$, $T_j = 1000j(1-0.01(j-1))$. Table 1 shows the computed results after applying Algorithm 1. The optimal solution occurs when $j = 3$, optimal ordering quantity for each RDC is $Q_r^{**} = 13557$, and the total network cost is $TNC^* = 3.15316 \times 10^6$. Eqs. (2) and (3) can then show that the optimal influence area for each RDC in cluster C_1 is $A_{r_1}^* = 4173.75$, the optimal influence area for each RDC in cluster C_2 is $A_{r_2}^* = 5022.37$, the optimal influence area for each RDC in cluster C_3 is $A_{r_3}^* = 6252.02$, and the ordering quantity for NDC is $Q_n^* = 3031$. Fig. 3 shows the graphic illustrations of TNC versus different decision variables.

$$H_i = \begin{bmatrix} \frac{\partial^2 TNC}{\partial A_{r_1}^2} & 0 & \cdot & \cdot & \cdot & \cdot & 0 & 0 \\ 0 & \frac{\partial^2 TNC}{\partial A_{r_2}^2} & \cdot & & & & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & & & \cdot & \cdot \\ \cdot & & \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & & & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & & & & \cdot & \cdot & 0 & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & 0 & \frac{\partial^2 TNC}{\partial A_{r_N}^2} & 0 \\ 0 & 0 & \cdot & \cdot & \cdot & \cdot & 0 & \frac{\partial^2 TNC}{\partial Q_n^2} \end{bmatrix}$$

Table 1
Result of the algorithm.

j	Q_r^*	B_j	TNC
1	–	5000	3.18911×10^6
2	–	10,000	3.15264×10^6
*3	13,557	–	3.15316×10^6
4	15,605	–	3.16303×10^6
5		25,000	3.16563×10^6

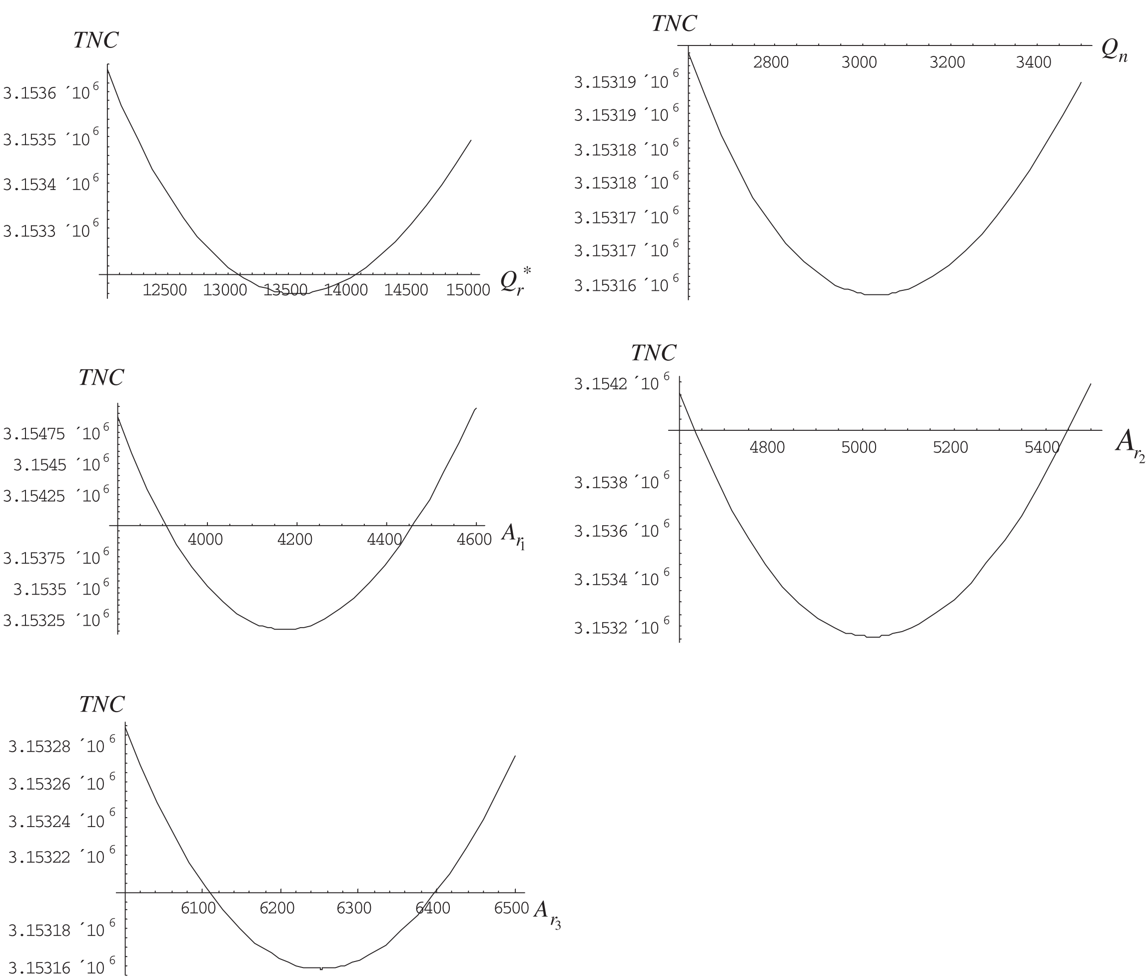

Fig. 3. Graphic illustrations of *TNC* versus different decision variables.

Table 2
Influence of freight cost $T_j = a \times j \times [1 - b(j-1)]$, with different a and b.

a	b		
	0.01	0.02	0.03
500	$Q_r^{**} = 7874$ $Q_n^* = 3031$ $A_{r_1}^* = 4099.36$ $A_{r_2}^* = 4932.86$ $A_{r_3}^* = 6140.60$ $TNC^* = 3.12526 \times 10^6$	$Q_r^{**} = 7835$ $Q_n^* = 3031$ $A_{r_1}^* = 4098.85$ $A_{r_2}^* = 4932.24$ $A_{r_3}^* = 6139.83$ $TNC^* = 3.12507 \times 10^6$	$Q_r^{**} = 7795$ $Q_n^* = 3031$ $A_{r_1}^* = 4098.33$ $A_{r_2}^* = 4931.62$ $A_{r_3}^* = 6139.05$ $TNC^* = 3.12487 \times 10^6$
1000	$Q_r^{**} = 13557$ $Q_n^* = 3031$ $A_{r_1}^* = 4173.75$ $A_{r_2}^* = 5022.37$ $A_{r_3}^* = 6252.02$ $TNC^* = 3.15316 \times 10^6$	$Q_r^{**} = 13417$ $Q_n^* = 3031$ $A_{r_1}^* = 4171.92$ $A_{r_2}^* = 5020.17$ $A_{r_3}^* = 6249.28$ $TNC^* = 3.15248 \times 10^6$	$Q_r^{**} = 13275$ $Q_n^* = 3031$ $A_{r_1}^* = 4170.06$ $A_{r_2}^* = 5017.94$ $A_{r_3}^* = 6246.5$ $TNC^* = 3.15179 \times 10^6$
1500	$Q_r^{**} = 19193$ $Q_n^* = 3031$ $A_{r_1}^* = 4246.86$ $A_{r_2}^* = 5110.35$ $A_{r_3}^* = 6361.54$ $TNC^* = 3.18011 \times 10^6$	$Q_r^{**} = 18887$ $Q_n^* = 3031$ $A_{r_1}^* = 4242.90$ $A_{r_2}^* = 5105.58$ $A_{r_3}^* = 6355.61$ $TNC^* = 3.17866 \times 10^6$	$Q_r^{**} = 18576$ $Q_n^* = 3031$ $A_{r_1}^* = 4238.88$ $A_{r_2}^* = 5100.75$ $A_{r_3}^* = 6349.58$ $TNC^* = 3.17719 \times 10^6$

5.2. Numerical analysis

This section chooses different values of a and b for T_j to reflect different freight costs and discount rates. Table 2 shows the influence of these various values on decision-making and cost. The results are as follows:

1. When a increases, the optimal ordering quantity for each RDC Q_r^{**}, the optimal influence area for each RDC $A_{r_i}^*$ and the total network cost TNC^* all increase. When the freight cost increases, it is reasonable for RDCs to increase their ordering quantity to reduce ordering frequency. Also, the company will increase each RDC's influence areas to reduce the number of RDCs. This effectively minimizes the total network cost.
2. When b increases, the optimal ordering quantity for each RDC Q_r^{**}, the optimal influence area for each RDC $A_{r_i}^*$ and the total network cost TNC^* all decrease. When the discount increases, it is reasonable to decrease RDC's ordering quantity and RDC's influence area, to take the advantage of quantity freight cost discounts more times.

It is also important to discuss the influence of the fixed transportation cost C_f and variable transportation cost C_v. This study includes a sensitivity analysis to further examine the effects of these two parameters. Table 3 provides the following management implications:

1. When C_f increases, the total network cost TNC^* increases. However, the optimal ordering quantity for each RDC Q_r^{**}, the optimal ordering quantity for NDC Q_n^*, and the optimal influence area for each RDC $A_{r_i}^*$ do not change as C_f increases. This means that the fixed transportation cost does not affect the optimal ordering quantities for each RDC, and that the optimal ordering quantity for NDC and the optimal influence area for each RDC. This verifies Property 1(c).
2. When C_v increases, the optimal ordering quantity for each RDC Q_r^{**} and the optimal influence area for each RDC $A_{r_i}^*$ decrease. However, the total network cost TNC^* increases. This verifies Property 1 (c). When the variable transportation cost increases, it is reasonable to decrease RDC's ordering quantity and RDC's influence area to reduce transportation costs.

The last experiment performed a sensitivity analysis to investigate the effects from ordering cost and inventory holding cost, including ordering costs for NDC and RDC (i.e. R_n and R_r) and inventory holding costs for NDC and RDC (i.e. h_n and h_r). This experiment was conducted by changing ordering costs up to ±67% and by changing inventory holding costs up to ±50%. Table 4 shows the numerical results. The results are as follows:

1. When the ordering cost for NDC R_n increases, the optimal ordering quantity for NDC Q_n^* and total network cost TNC^* both increase. When ordering cost for RDC R_r increases, the optimal ordering quantity for RDC Q_r^{**}, the optimal influence area for each RDC $A_{r_i}^*$, and the total network cost TNC^* all increase. If the ordering cost increases, it is reasonable that the company will increase the ordering quantity to reduce replenishment frequency. The company will also likely increase each RDC's influence areas to reduce the number of RDCs as the ordering cost increases.

Table 3
Influence of different C_f and C_v.

C_f	C_v		
	5	7.5	10
10	$Q_r^{**}=19787$ $Q_n^*=3031$ $A_{r_1}^*=6753.65$ $A_{r_2}^*=8126.83$ $A_{r_3}^*=10116.6$ $TNC^*=2.57291\times10^6$	$Q_r^{**}=14954$ $Q_n^*=3031$ $A_{r_1}^*=5078.16$ $A_{r_2}^*=6110.67$ $A_{r_3}^*=7606.78$ $TNC^*=2.87623\times10^6$	$Q_r^{**}=13557$ $Q_n^*=3031$ $A_{r_1}^*=4173.75$ $A_{r_2}^*=5022.37$ $A_{r_3}^*=6252.02$ $TNC^*=3.15316\times10^6$
50	$Q_r^{**}=19787$ $Q_n^*=3031$ $A_{r_1}^*=6753.65$ $A_{r_2}^*=8126.83$ $A_{r_3}^*=10116.6$ $TNC^*=8.69771\times10^6$	$Q_r^{**}=14954$ $Q_n^*=3031$ $A_{r_1}^*=5078.16$ $A_{r_2}^*=6110.67$ $A_{r_3}^*=7606.78$ $TNC^*=9.00103\times10^6$	$Q_r^{**}=13557$ $Q_n^*=3031$ $A_{r_1}^*=4173.75$ $A_{r_2}^*=5022.37$ $A_{r_3}^*=6252.02$ $TNC^*=9.27796\times10^6$
100	$Q_r^{**}=19787$ $Q_n^*=3031$ $A_{r_1}^*=6753.65$ $A_{r_2}^*=8126.83$ $A_{r_3}^*=10116.6$ $TNC^*=1.63537\times10^7$	$Q_r^{**}=14954$ $Q_n^*=3031$ $A_{r_1}^*=5078.16$ $A_{r_2}^*=6110.67$ $A_{r_3}^*=7606.78$ $TNC^*=1.66570\times10^7$	$Q_r^{**}=13557$ $Q_n^*=3031$ $A_{r_1}^*=4173.75$ $A_{r_2}^*=5022.37$ $A_{r_3}^*=6252.02$ $TNC^*=1.69340\times10^7$

Table 4
The effects from ordering cost and inventory holding cost.

Parameter	Q_r^{**}	Q_n^*	$A_{r_1}^*$	$A_{r_2}^*$	$A_{r_3}^*$	TNC^*
$R_n = 10$	13,557	1750	4173.75	5022.37	6252.02	3.15188×10^6
$R_n = 30$	13,557	3031	4173.75	5022.37	6252.02	3.15316×10^6
$R_n = 50$	13,557	3913	4173.75	5022.37	6252.02	3.15401×10^6
$R_r = 10$	13,511	3031	4173.17	5021.64	6251.11	3.15293×10^6
$R_r = 30$	13,557	3031	4173.75	5022.37	6252.02	3.15316×10^6
$R_r = 50$	13,604	3031	4174.35	5023.10	6252.93	3.15338×10^6
$h_n = 0.5$	13,557	4287	4173.75	5022.37	6252.02	3.15227×10^6
$h_n = 1$	13,557	3031	4173.75	5022.37	6252.02	3.15316×10^6
$h_n = 1.5$	13,557	2475	4173.75	5022.37	6252.02	3.15384×10^6
$h_r = 0.9$	14,275	3031	4164.49	5011.23	6238.16	3.14971×10^6
$h_r = 1$	13,557	3031	4173.75	5022.37	6252.02	3.15316×10^6
$h_r = 1.1$	12,904	3031	4182.56	5032.97	6265.22	3.15643×10^6

2. When the inventory holding cost for NDC h_n increases, the optimal ordering quantity for NDC Q_n^* decreases but the total network cost TNC^* increases. This verifies Property 1(b). When the inventory holding cost for RDC h_r increases, the optimal ordering quantity for RDC Q_r^{**} decreases, while the optimal influence area for each RDC $A_{r_i}^*$ and the total network cost TNC^* both increase. It is reasonable that when the inventory holding cost increases, the company will decrease the ordering quantity in an effort to lower inventory costs. They will increase each RDC's influence areas to reduce the number of RDCs as the inventory holding cost increases.

6. Extension: Different ordering quantity for RDC in different cluster

This section relaxes Assumption 5 to consider different ordering quantities for RDCs in different clusters and determine the difference between equal ordering quantity and different ordering quantity. First, consider the following notations:

Q_{r_i}	ordering quantity for RDC in cluster C_{n_i}, where $i = 1,2,\ldots,N$
$T_{i,j}$	inbound transportation cost per item for $Q_{r_i},\ B_j < Q_{r_i} \leqslant B_{j+1}$, where $T_{i,j} < T_{i,j+1}$ and $T_{i,j}/B_j > T_{i,j+1}/B_{j+1}$, $j = 1,2,\ldots,n$

The total network cost is

$$TNC(A_{r_i},Q_n,Q_{r_i}) = \sum_{i=1}^{N}\left(F_r\frac{C_{n_i}}{A_{r_i}}\right) + \sum_{i=1}^{N}\left(T_{i,j}\frac{\xi\lambda_i\delta_iC_{n_i}}{Q_{r_i}}\right) + \sum_{i=1}^{N}\left[(C_f + C_v f_r\sqrt{A_{r_i}})(\xi\lambda_i\delta_iC_{n_i})\right] + R_n\frac{\sum_{i=1}^{N}(\xi\lambda_i\delta_iC_{n_i})}{Q_n} + \sum_{i=1}^{N}\left(R_r\frac{\xi\lambda_i\delta_iC_{n_i}}{Q_{r_i}}\right) + h_n\left(\frac{Q_n}{2} + Z_\alpha\sqrt{\sum_{i=1}^{N}\mu\lambda_i\delta_iC_i}\right) + \sum_{i=1}^{N}\left(h_r\frac{C_{n_i}}{A_{r_i}}\frac{Q_{r_i}}{2}\right). \tag{6}$$

The problem here is to determine A_{r_i}, Q_n, Q_{r_i}, $i = 1, 2, \ldots, N$, to minimize the total network cost. Given $Q_{r_i},\ i = 1,2,\ldots,N$, we have

$$\frac{\partial^2 TNC(A_{r_i},Q_n|Q_{r_i})}{\partial A_{r_i}^2} = \sum_{i=1}^{N}\left(\frac{2F_rC_{n_i}}{A_{r_i}^3}\right) + \sum_{i=1}^{N}\left(\frac{-C_vf_r\xi\lambda_i\delta_iC_{n_i}}{4A_{r_i}^{3/2}}\right) + \sum_{i=1}^{N}\left(\frac{h_rC_{n_i}Q_{r_i}}{A_{r_i}^3}\right), \quad i = 1,2,\ldots,N,$$

$$\frac{\partial^2 TNC(A_{r_i},Q_n|Q_{r_i})}{\partial Q_n^2} = 2R_n\frac{\sum_{i=1}^{N}(\xi\lambda_i\delta_iC_{n_i})}{Q_n^3} > 0,$$

$$\frac{\partial^2 TNC(A_{r_i},Q_n|Q_{r_i})}{\partial A_{r_i}\partial A_{r_j}} = 0, j = 1,2,\ldots,N, \quad \text{but } i \neq qj,$$

$$\frac{\partial^2 TNC(A_{r_i},Q_n|Q_{r_i})}{\partial A_{r_i}\partial Q_n} = 0.$$

Since the facility opening cost F_r is large, $\frac{\partial^2 TNC(A_{r_i},Q_n|Q_{r_i})}{\partial A_{r_i}^2} > 0$ is satisfied in the general case. The Hessian matrix is similar to that in Section 3. The minimum theorem in Winston (2004) indicates that the optimal A_{r_i} and Q_n can be obtained by solving $\frac{\partial TNC(A_{r_i},Q_n|Q_{r_i})}{\partial A_{r_i}} = 0$ and $\frac{\partial TNC(A_{r_i},Q_n|Q_{r_i})}{\partial Q_n} = 0$

$$A_{r_i}(Q_{r_i}) = \left(\frac{2F_r + h_rQ_{r_i}}{C_vf_r\xi\lambda_i\delta_i}\right)^{2/3}, \tag{7}$$

$$Q_n = \sqrt{\frac{2R_n\sum_{i=1}^{N}\xi\lambda_i\delta_iC_{n_i}}{h_n}}. \tag{8}$$

Eqs. (7) and (8) can then obtain Property 2.

Property 2.

(a) *The influence area for each RDC A_{r_i} increases as the ordering quantity for RDC Q_{r_i} or the facility cost of opening each RDC F_r increase.*
(b) *The ordering quantity for NDC Q_n increases as the inventory holding cost for NDC h_n decreases or the ordering cost for NDC R_n increases.*
(c) *The influence area for each RDC A_{r_i} decreases as the variable transportation cost C_v increases. However, A_{r_i} does not change as C_f changes.*

Substituting Eqs. (7) and (8) into Eq. (6) yields

$$TNC(Q_{r_1},Q_{r_2},\ldots,Q_{r_N}) = \sum_{i=1}^{N}\left[F_rC_{n_i}\left(\frac{C_vf_r\xi\lambda_i\delta}{2F_r+h_rQ_{r_i}}\right)^{2/3}\right]+\sum_{i=1}^{N}\left\{\left[C_f+C_vf_r\left(\frac{2F_r+h_rQ_{r_i}}{C_vf_r\xi\lambda_i\delta_i}\right)^{1/3}\right](\xi\lambda_i\delta_iC_{n_i})\right\}$$
$$+\sum_{i=1}^{N}\left[\frac{h_rC_{n_i}Q_{r_i}}{2}\left(\frac{C_vf_r\xi\lambda_i\delta}{2F_r+h_rQ_{r_i}}\right)^{2/3}\right]+\sqrt{2h_nR_n\sum_{i=1}^{N}(\xi\lambda_i\delta_iC_{n_i})}+h_nZ_\alpha\sqrt{\sum_{i=1}^{N}\mu\lambda_i\delta_iC_i}$$
$$+\sum_{i=1}^{N}\left(R_r\frac{\xi\lambda_i\delta_iC_{n_i}}{Q_{r_i}}\right)+\sum_{i=1}^{N}\left(T_{i,j}\frac{\xi\lambda_i\delta_iC_{n_i}}{Q_{r_i}}\right). \quad (9)$$

This study provides an heuristic algorithm based on a loop and Algorithm 1 to obtain the optimal ordering quantity $Q^{**}_{r_i},\ i=1,2,\ldots,N$.

Algorithm 2.

Step 1. Given $Q_{r_2},\ldots,Q_{r_N}$ and start with $i = 1$.
 1.1: Beginning with the freight cost $T_{i,j=1}$, search for the order quantity Q_{r_i} that minimizes Eq. (9) until a valid optimal $Q^*_{r_i}$ is found (i.e., $Q^*_{r_i}$ must fall within the corresponding break quantity range, $B_{i,j-1} < Q_{r_i} \leqslant B_{i,j}$) or $j = n$.
 1.2. If a valid optimal $Q^*_{r_i}$ is found, let the corresponding break quantity range be $B_{i,j^*-1} < Q \leqslant B_{i,j^*}$ and go to Step 2.1; otherwise go to Step 2.2.
Step 2: Select the optimal order quantity for each RDC in cluster C_{n_i}.
 2.1: Compare the total network cost $TNC(Q^*_{r_i})$ and $TNC(B_{i,j})$ with all $j < j^*$. Select the value ($Q^*_{r_i}$ or $B_{i,j}$) that minimizes the total network cost and let $Q^{**}_{r_i}$ be the optimal value; then go to Step 3.
 2.2: Compare the annual profit $TNC(B_{i,j})$ for all $j \leqslant n$. Select the value $B_{i,j}$ that minimizes the total network cost and let $Q^{**}_{r_i}$ be the optimal value; then go to Step 3
Step 3: If $i < N$, let $i = i + 1$, go to Step 1.1; otherwise go to Step 4.
Step 4: Let $TNC^{**} = TNC(Q^{**}_{r_1}, Q^{**}_{r_2},\ldots,Q^{**}_{r_N})$.

This section follows the same data in experiment in Section 4.1. After applying Algorithm 2, the optimal solution is when $j = 3$ for all clusters, optimal ordering quantity for RDC in cluster C_{n_1} is $Q^{**}_{r_1} = 14,023$, optimal ordering quantity for RDC in cluster C_{n_2} is $Q^{**}_{r_2} = 13,375$, optimal ordering quantity for RDC in cluster C_{n_3} is $Q^{**}_{r_3} = 12,648$, and the total network cost is $TNC^* = 3.15311 \times 10^6$. Then, Eqs. (7) and (8) show that the optimal influence area for each RDC in cluster C_{n_1} is $A^*_{r_1} = 4179.81$, the optimal influence area for each RDC in cluster C_{n_2} is $A^*_{r_2} = 5019.52$, the optimal influence area for each RDC in cluster C_{n_3} is $A^*_{r_3} = 6234.27$, and the ordering quantity for NDC is $Q^*_n = 3031$. The total network cost *TNC* are very close to those considering that the order quantity Q_r is the same across all RDCs. When focusing on the total network cost, this concurs with the assumption of Deuermeyer and Schwarz (1981) and Ganeshan (1999): it is a common practice to assume that the order quantity Q_r is the same across all RDCs.

7. Conclusions

This study uses a novel two-phase approximation method to solve the multi-echelon supply network design problem. The proposed method integrates facility costs, inventory costs, transportation costs, and ordering costs. This method makes it possible to solve problems with huge amounts of non-homogeneous demand data. The key decisions are where to locate the RDCs, how to assign retail stores to RDCs, and how to set the inventory policy at different locations to minimize the total network cost. Numerical studies illustrate the solution procedures and the impacts of the related parameters on decisions and costs. The proposed model provides a powerful analysis tool for studying potential changes in a supply chain system due to changes in the parameters. This study also solves the problem of different ordering quantities for each RDC in different cluster. The results of this study are a useful reference for managerial decision-making and administration.

Our research contributes to the literature (Daskin et al., 2002; Shen et al., 2003; Teo and Shu, 2004; Romeijn et al., 2007; Shen and Qi, 2007; Mangotra et al., 2009) in several ways. The contributions of this paper to the literature are as follows. First, this is the first study based on continuous approximation approach for supply chain network design problem with transportation cost discounts and inventory decisions. In particular, this study simultaneously considers two common types of transportation cost discounts (quantity discounts and distance discounts). Second, the proposed solution defines the input data in terms of continuous functions and is capable of formulating these functions for a data set of any size. This is very important for dealing with practical problems. Third, this study proposes an efficient heuristic method for solving resulting nonlinear programs. We also conduct numerical analysis to discuss impacts of the changing parameters and provide the management implications. Fourth, this article considers different ordering quantities for RDCs in different regions. An iterative solution procedure is proposed to solve the problem. We believe this paper provides a good starting point in this research stream. Further research on this topic could relax some assumptions to match real-world scenarios, such as capacity limitations on DCs and multiple products.

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