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Formulating the multi-segment goal programming

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ABSTRACT

Goal programming (GP) is an important analytical approach devised to solve many real-word problems. However, the condition of multi-segment aspiration levels (MSAL) may exist in many marketing or decision management problems. The problem cannot be solved by current GP techniques. In order to improve the effective of GP and solve the multi-segment goal programming (MSGP) problem, this paper provides a new idea for programming the MSAL problem from multi-aspiration contribution levels viewpoint. This significantly improved the utility of GP in real application; in addition, two illustrative examples are included to demonstrate the solution procedure of the proposed model.

1. Introduction

Goal programming (GP) is a multi-objectives analytical approach devised to address decision-marking problems where targets have been assigned to all attributes and where the decision-makers (DMs) are interested in minimizing the non-achievement of the corresponding goal. The model allows taking into account simultaneously many objectives while the decision-marking is seeking the best solution from among a set of feasible solutions. GP was first introduced by Charnes and Cooper (1961), and further developed by Lee (1972), Ignizio (1976), Tamiz, Jones, and Romero (1998), Romero (2001), Chang (2004); among others. The oldest form can be expressed as follows: (*GP* model)

$$\text{Minimize } \sum_{i=1}^{n} |f_i(X) - g_i|$$

Subject to $X \in F(F \text{ is a feasible set})$,

where $f_i(X)$ is the linear function of the ith goal, g_i and is the aspiration level of the ith goal.

The above minimization process can be accomplished with various types of methods such as weighted GP (WGP), lexicographic GP (LGP), and Ckebyshev or MINMAX GP (MGP). The three oldest and still most widely used forms of GP achievement functions are the following (Romero, 2001):

(i) The achievement function of WGP model lists the unwanted deviation variables, each weighted according to importance. The mathematical formulations of a WGP model is the following (Ignizio, 1976):
(*WGP* model)

Achievement function:

$$\text{Minimize } \sum_{i=1}^{n} (\alpha_i d_i^+ + \beta_i d_i^-).$$

Goals and constraints:

$$\text{Subject to } f_i(X) + d_i^- - d_i^+ = g_i, \quad i = 1, 2, \ldots, n,$$
$$d_i^+, d_i^- \geqslant 0, \quad i = 1, 2, \ldots, n,$$
$$X \in F(F \text{ is a feasible set}),$$

where parameter α_i and β_i are the weights reflecting preferential and normalizing purposes attached to positive and negative deviations of ith goal, respectively; $d_i^- = \max(0, g_i - f_i(X))$, $d_i^+ = \max(0, f_i(X) - g_i)$ are, respectively, under- and over-achievements of the ith goal; $f_i(X)$ and g_i are defined as in GP model.

(ii) The achievement function of LGP model is made up of an ordered vector whose dimension coincides with the Q number of priority levels established in the model. Each component in this vector represents the unwanted deviation variables of goal placed in the corresponding priority level. The mathematical formulations of a LGP model is the following (Ignizio, 1976):

(*LGP* model)

Achievement function:

Let minimize a

$$= \left[\sum_{i \in h_i} (\alpha_i d_i^- + \beta_i d_i^+), \ldots, \sum_{i \in h_r} (\alpha_i d_i^- + \beta_i d_i^+), \ldots, \sum_{i \in h_Q} (\alpha_i d_i^- + \beta_i d_i^+) \right]$$

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Goals and constraints:

$$\text{Subject to } f_i(X) + d_i^- - d_i^+ = g_i, \quad i = 1,2,\ldots,n, \quad i \in h_r$$
$$r = 1,2,\ldots,Q,$$
$$d_i^+, d_i^- \geqslant 0, \quad i = 1,2,\ldots,n,$$
$$X \in F(F \text{ is a feasible set}),$$

where h_r represents the index set of goals placed in the rth priority level; other variables are defined as in WGP model.

(iii) The achievement function of MGP model implies the minimization of the maximum deviation from any single goal. Furthermore, as some situation hold corresponding solution portray a balanced allocation among the achievement of the different goals (Romero, 2001). If D is an extra continuous variable that measures the maximum deviation; this maximum deviation the mathematical formulations of a MGP model is the following (Flavell, 1976):
(*MGP* model)
Achievement function:

$$\text{Minimize } D$$

Goals and constraints:

$$\text{Subject to } D \geqslant \alpha_i d_i^+ + \beta_i d_i^-,$$
$$f_i(X) - d_i^- + d_i^+ = g_i, \quad i = 1,2,\ldots,n,$$
$$d_i^+, d_i^- \geqslant 0, \quad i = 1,2,\ldots,n,$$
$$X \in F(F \text{ is a feasible set}),$$

where other variables are defined as in WGP model.
While, the WGP, LGP and MGP models of the achievement function are the most widely used, other extensions may represent under certain situations the MDs preferences with more accuracy. There are another formulations structure can be presented as follows:
(*EGP* model)

Achievement function:

$$\text{Minimize } (1-\lambda)D + \lambda \sum_{i=1}^{n} (\alpha_i d_i^+ + \beta_i d_i^-)$$

Goals and constraints:

$$\text{Subject to } \alpha_i d_i^+ + \beta_i d_i^- - D \leqslant 0,$$
$$f_i(X) - d_i^- + d_i^+ = g_i, \quad i = 1,2,\ldots,n,$$
$$d_i^+, d_i^- \geqslant 0, \quad i = 1,2,\ldots,n,$$
$$X \in F(F \text{ is a feasible set}),$$

where parameter λ weights the importance attached to the minimization of the weighted sum of unwanted deviation variables. For $\lambda = 0$, we have the MGP achievement function, for $\lambda = 1$ the WGP achievement function and for other values of parameter λ belonging to the interval $(0,1)$ intermediate solutions provided by the weighted combination of these two GP models options (Romero, Tamiz, & Jones, 1998). Then, other variables are defined as in WGP model.
In addition, for reducing the number of additional variables (i.e., d_i^- and d_i^+) used in WGP where $\alpha_i = 1$ and $\beta_i = 1$. Li (1996) proposed an equivalent approach as follows:
Achievement function:

$$\text{Minimize } \sum_{i=1}^{n} 2d_i - f_i(X) + g_i$$

Goals and constraints:

$$\text{Subject to } -f_i(X) + d_i + g_i \geqslant 0, \quad i = 1,2,\ldots,n,$$
$$d_i \geqslant 0, \quad i = 1,2,\ldots,n,$$
$$X \in F(F \text{ is a feasible set}),$$

where the positive deviation and negative of the ith goal are d_i and $-f_i(X) + g_i$, respectively.

These approaches have been applied to solve many real-world problems (Tamiz et al., 1998). However, in some condition, such as where a DM would like make a decision on the problem, which involved the achievement of goal, some of them are met or not met (see e.g., Chang, 2004). This problem cannot be solved by above GP approach. In order to solve which involved the achievement of goals, some of MDs are met and some are not met decision/management problem. Chang (2004) presented a mixed binary GP (MBGP) method to solve the problems, and the expressed as follows:

Achievement function:

$$\text{Minimize } \sum_{i=1}^{n} (d_i^+ + d_i^-) b_i$$

Goals and constraints:

$$\text{Subject to } (f_i(X) - g_i) b_i + d_i^- - d_i^+ = 0, \quad i = 1,2,\ldots,n,$$
$$d_i^+, d_i^- \geqslant 0, \quad i = 1,2,\ldots,n,$$
$$b_i \in R_i, \quad i = 1,2,\ldots,n, \quad X \in F(F \text{ is a feasible set}),$$

where R_i is the environment constraint function of ith goal; b_i is the binary variable of ith goal.

One common characteristic of all the different types of GP models introduced so far (including WGP, LGP, MGP, EGP and MBGP) each goal is formulated in a precise way with coefficients defined by crispy numbers. This implies that all managerial objectives for the problem being studied can be encompassed within only a single goal. However, this is not always associated with certain attributes in real-life. Often, in real-world problems the objectives are imprecise (or fuzzy) (Gen, Ida, Lee, & Kim, 1997). For example, many imprecise aspiration levels may exist such as "some what larger than", "substantially lesser than", or "around" the vague goal. In doing so, if the imprecise aspiration level is introduced to each goal of GP, the problem is then turned to fuzzy GP (FGP) (Zimmermann, 1978). Other example, many multi-choice aspiration levels may exist such as "something more/higher is better in the aspiration levels", or "something less/lower is better in the aspiration levels" the multi-choice goal. Following the idea of FGP theory, the problem is then turned to multi-choice GP (MCGP) (Chang, 2007).

Although the FGP and MCGP functions offer a simple concept for the vague phenomena or multi-choice in goal levels, the important area of decision variables coefficients analysis (e.g., the different contribution levels of decision variable coefficients, or multi-segment aspiration levels) is still open. For example, companies/organizations often adjust their basic price to accommodate differences in customer, products, locations, and so on. Such as museums often change a lower admission fee to students and senior citizens, and wireless telecommunication industry utilities vary energy rates to commercial users by time of day and weekend versus weekday (Kotler & Keller, 2006). This is a multi-segment GP (MSGP) problem. One of the characteristics of a FGP or MBGP model is that the decision variables are allowed to have only one value-level, which satisfy the various constraints. The purpose of this work is to derive a new approach for solving the MODM problem with MSAL. The proposed idea for solving the MODM problem with MSAL is very different from GP using membership function to manage the MODM problem with imprecise aspiration levels of the decision variables coefficients, we say, multi-segment aspiration levels.

The remainder of this paper is organized as follows. Section 2 introduces the multi-segment GP (MSGP) formulation. In order to demonstrate the correctness of the proposed model, illustrative examples are included in Section 3. Finally, Section 4 summarizes and points towards directions for future research.

2. Multi-segment GP model

When managing a for-profit organization, the managerial objective might well include some of the following; obtain distinction profits, increase marketing share, diversify the product line, segment the product price etc., in different market. In fact, the conflicts of organizational resource and the incompleteness of available information make if almost impossible for DMs to build a reliable mathematical model for representation of their preference (Chang, 2007). The objectives are so different in organization that it really is not realistic to combine them into a single precise goal. In order solve the problem of MSAL, the DMs attempt to set a goal to get the acceptable solutions in which DMs would interest to minimize the deviations between the achievements of goal and their aspiration levels of decision variable coefficients. To the best knowledge of our, no work has been done for solving this typical MSGP problem. The MSGP can be expressed as follows:

Achievement function:

$$\text{Minimize } z = \{g_1(d_1^+, d_1^-), g_2(d_2^+, d_2^-), \ldots, g_n(d_n^+, d_n^-)\}$$

Goals and constraints:

$$\begin{aligned} \text{Subject to } & \sum_{i=1}^{n} s_{ij}X_i + d_i^- - d_i^+ = g_i, \quad j = 1,2,\ldots,m \\ & s_{ij} = s_{i1} \text{ or } s_{i2} \text{ or } \ldots \text{ or } s_{im}, \quad i = 1,2,\ldots,n, \\ & X \in F(F \text{ is a feasible set}), \end{aligned}$$

where s_{ij} is a decision variable coefficients, represents the multi-segment aspiration levels of jth segment of ith goal; other variables are defined as in WGP.

For detailed descriptions of the MSGP problem, we depicted three cases of decisions problem in Figs. 1–3, respectively. Without loss of generality, we used WGP model and let $\alpha_i = 1$ and $\beta_i = 1$, therefore, three cases should be considered as follows:

A	B	C
S_{11}	S_{21}	S_{31}

Fig. 1. Example of MSGP (one-segment aspiration level).

A	B	C
S_{11}	S_{21}	S_{31}
S_{21}	S_{22}	S_{23}

Fig. 2. Example of MSGP (two-segment aspiration levels).

A	B	C
S_{11}	S_{21}	S_{31}
S_{12}	S_{22}	S_{23}
S_{13}	S_{23}	S_{33}

Fig. 3. Example of MSGP (multi-segment aspiration levels).

(i) If only one-segment aspiration level in each market. For example, there are three aspiration contribution levels s_{11}, s_{21}, and s_{31} corresponding to market A, B, and C (see Fig. 1). This case is a traditional MODM problem that it can be formulated using WGP as described below.

$$\text{Minimize} \sum_{i=1}^{3}(d_i^+ + d_i^-)$$

$$\begin{aligned} \text{Subject to} & \sum_{i=1}^{3} s_{ij}x_i + d_i^- - d_i^+ = g_i, \quad i = 1,2,3, \quad j = 1,2,3, \\ & d_i^+, d_i^- \geqslant 0, \quad i = 1,2,3, X \in F(F \text{ is a feasible set}), \end{aligned}$$

where all variables are defined as in MSGP.

(ii) If only two-segment aspiration levels in each market. This is a case of MODM problem with an either-selection. The aspiration contribution level in segment A is to select an appropriate level from either s_{11} or s_{12}, while the aspiration contribution level in segment B is to select an appropriate level from either s_{21} or s_{22}, and in segment C is to select s_{31} or s_{32}, similarly, as depicted in Fig. 2. This case cannot be solved by current GP approaches. Following the logic of the Chang (2007) in developing MCGP model, in order to solve the problem, three extra binary variables should be added as described below.

$$\text{Minimize} \sum_{i=1}^{3}(d_i^+ + d_i^-)$$

$$\begin{aligned} \text{Subject to} & (s_{11}b_1 + s_{12}(1-b_1))x_1 + s_{21}x_2 + s_{31}x_3 + d_1^- - d_2^+ = g_1, \\ & s_{11}x_1 + (s_{21}b_2 + s_{22}(1-b_2))x_2 + s_{31}x_3 + d_2^- - d_2^+ = g_2, \\ & s_{11}x_1 + s_{21}x_2 + (s_{31}b_3 + s_{32}(1-b_3))x_3 + d_3^- - d_3^+ = g_3, \\ & d_i^+, d_i^- \geqslant 0, \quad i = 1,2,3, \\ & X \in F(F \text{ is a feasible set}), \end{aligned}$$

where b_1, b_2 and b_3 are binary variables; other variables are defined as in MSGP.

(iii) If multi-segment aspiration levels in each market. This case is a multi-selection MODM problem. The aspiration contribution level in segment A is to select an appropriate level from either s_{11}, s_{12}, or s_{13}, while the aspiration contribution level in segment B is to select an appropriate level from either s_{21}, s_{22}, or s_{23}, and in segment C is to select s_{31}, s_{32}, or s_{33} similarly, as depicted in Fig. 3. This case cannot be solved by current GP approaches. Following the logic of the Chang (2007) in developing MCGP model, in order to solve the problem, six extra binary variables should be added as described below.

$$\text{Minimize} \sum_{i=1}^{3}(d_i^+ + d_i^-)$$
$$\text{Subject to } (s_{11}b_1b_2 + s_{12}b_1(1-b_2) + s_{13}(1-b_1)b_2)x_1 + s_{21}x_2 + s_{31}x_3 + d_1^- - d_2^+ = g_1,$$
$$s_{11}x_1 + (s_{21}b_3b_4 + s_{22}b_3(1-b_4) + s_{23}(1-b_3)b_4)x_2 + s_{31}x_3 + d_2^- - d_2^+ = g_2,$$
$$s_{11}x_1 + s_{21}x_2 + (s_{31}b_5b_6 + s_{32}b_5(1-b_6) + s_{33}(1-b_5)b_6)x_3 + d_3^- - d_3^+ = g_3,$$
$$d_i^+, d_i^- \geqslant 0,\ i = 1,2,3,$$
$$X \in F(F \text{ is a feasible set}),$$

where b_1, b_2, b_3, b_4, b_5 and b_6 are binary variables; other variables are defined as in MSGP.

The quadratic binary terms b_1b_2, b_3b_4 and b_5b_6 can be linearized (Chang, 2007). We assume that $h = b_i\, b_j$, where h satisfy the following inequalities:

$$(b_i + b_j - 2) + 1 \leqslant h \leqslant (2 - b_i - b_j) + 1, \tag{1}$$
$$h \leqslant b_i, \tag{2}$$
$$h \leqslant b_j, \tag{3}$$
$$h \geqslant 0. \tag{4}$$

The above inequalities can be checked as follows:

(i) if $b_i = b_j = 1$ then $h = 1$ (from (1)).
(ii) if $b_ib_j = 0$ then $h = 0$ (from (2)–(4)).

3. Some illustrative examples

In this section two examples will be used in order to illustrate the MSGP problem with the following multi-aspiration segment levels (decision variable coefficients) and constraint, which cannot be solved by current GP approaches.

Example 1 (*segment*).

$$\text{Goals}: (g_1)\ (3 \text{ or } 6)x_1 + 2x_2 + x_3 = 115,$$
$$(g_2)\ 4x_1 + (5 \text{ or } 9)x_2 + 2x_3 = 80,$$
$$(g_3)\ 3.5x_1 + 5x_2 + (7 \text{ or } 10)x_3 = 110.$$

Constraints:

$$x_2 + x_3 \geqslant 9, \quad x_2 \geqslant 5, \quad x_1 + x_2 + x_3 \geqslant 21.$$

where assumed decision variable coefficients denoted product price; x_1, x_2, and x_3 represents three products, and target values (e.g., 115, 80 and 110) are three markets profit goal, respectively.

Based on the MSGP method, this problem can be formulated as the following program:

$$\text{Minimize } z = d_1^+ + d_1^- + d_2^+ + d_2^- + d_3^+ + d_3^-$$
$$\text{Subject to } (3b_1 + 6(1-b_1))x_1 + 2x_2 + x_3 + d_1^+ - d_1^- = 115,$$
$$4x_1 + (5b_2 + 9(1-b_2))x_2 + 2x_3 + d_2^+ - d_2^- = 80,$$
$$3.5x_1 + 5x_2 + (7b_3 + 10(1-b_3))x_3 + d_3^+ - d_3^- = 110,$$
$$x_2 + x_3 \geqslant 9, \quad x_2 \geqslant 5, \quad x_1 + x_2 + x_3 \geqslant 21,$$
$$d_i^+, d_i^- \geqslant 0 \quad (i = 1,2,3),$$

where b_1, b_2, and b_3 are binary variables; d_i^+ and d_i^- are the positive and negative deviation variables, respectively.

Solve this MSGP problem using LINGO (Schrage, 1999) to obtain the optimal solutions as $(x_1, x_2, x_3, b_1, b_2, b_3) = (11.5\bar{4}, 5.0\bar{0}, 4.4\bar{6}, 0, 1, 0)$. From the results we realize that goal g_1 has $83.7\bar{0}$ achieved reached the aspiration level 115, goal g_2 has $73.6\bar{0}$ achieved reached the aspiration level 80, and goal g_3 has $109.8\bar{5}$ achieved reached the aspiration level 110.

Moreover, let us consider a MODM problem. It is slightly modified from Example 1.

Example 2 (*no segment*).

$$\text{Goals}: (g_1)\ 3x_1 + 2x_2 + x_3 = 115,$$
$$(g_2)\ 4x_1 + 9x_2 + 2x_3 = 80,$$
$$(g_3)\ 3.5x_1 + 5x_2 + 7x_3 = 110.$$

Constraints:

$$x_2 + x_3 \geqslant 9, x_2 \geqslant 5, x_1 + x_2 + x_3 \geqslant 21.$$

where the decision variable coefficients, variables and target values are defined the same as in Example 1.

Formulate this MODM problem using WGP and then solve it by LINGO (Schrage, 1999) to obtain the optimal solutions as $(x_1, x_2, x_3) = (7.7\bar{1}, 5.0\bar{0}, 8.2\bar{9})$. From the results we realize that goal g_1 has a negative value $(-73.5\bar{7})$ under aspiration level 115, goal g_2 has a positive value $(+12.4\bar{3})$ over aspiration level 80, and goal g_3 has $110.0\bar{2}$ achieved reached the aspiration level 110.

From the result, we realize that the solution of Example 1 is better than of Example 2 for a DM, because the solution of Example is indeed balanced on the three goals. Therefore, the more the aspiration contribution levels the better the solutions found in the proposed MSGP method.

4. Conclusions

Marketing decision making such as price discrimination, customer segment, time segment, location and channel segment designs are often formulated as multi-segment aspiration level problems. This is a case of multi-segment MODM problem. To the best knowledge of our, this problem cannot be solved by current GP approaches. This paper proposes a new formulation method for solving the MSGP problem, which cloud certainly obtains a solution close to the DMs multi-segment aspiration levels. Consequently, the practical utility of GP approach has been expanded in this paper.

The promising results stimulate the need for future research on nonlinear function of the aspiration levels. Preemptive nonlinear goal programming (NGP) is a mathematical programming technique for solving multiple criteria mathematical programming problems involving nonlinear objectives and nonlinear constraint (Zheng, Gen, & Ida, 1996).

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