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Negative Priorities in the Analytic Hierarchy Process

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Abstract—In decision-making, there are often criteria that are opposite in direction to other criteria as in benefits (B) versus costs (C), and in opportunities (O) versus risks (R), and sometimes need to be distinguished by using negative numbers. In making paired comparisons of alternatives with respect to a benefits criterion, one always uses the fundamental scale of positive absolute values of the analytic hierarchy process to estimate how much more benefits an alternative yields than the another alternative with which it is compared, puts the final values in the idealized mode of the AHP and synthesizes the results for the criteria under benefits. One does the same for a costs criterion to determine how much more one alternative costs than another, forms the ideal and synthesizes for the costs criteria. Similarly for opportunities and risks. One then needs to combine the four sets of priorities to get the overall ranking of the alternatives. Several different ways are described in the paper for doing this. A fundamental problem in the process of combining the B, O, C, and R had to be solved first and was done by the first author using ratings rather than paired comparisons in an earlier work done in 1999 described in this paper and used to deal with combining priorities that are opposite in direction. It is pointed out in the paper that each of the positive or negative priorities need not have a symmetric opposite value, because the opposite criterion may not exist in practice.

Keywords—Analytic hierarchy process, Benefits-costs-opportunities-risks, Decision making, Negative priorities.

1. INTRODUCTION

The analytic hierarchy process (AHP) is a discrete measurement theory that derives scales of values from pairwise comparisons and from ratings [1]. The comparisons use the smaller or lesser of a pair of elements as the unit with respect to a given property, and estimate the number of multiples of that unit the larger or greater element is. In ratings, intensities (e.g., excellent, very good, good, average, poor; or simply high, medium, low) are pairwise compared with respect

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to each criterion and each alternative is then assigned one of these intensities for that criterion. In both paired comparisons and ratings, the numbers used are positive and the numbers in the scales derived from them are also positive, and further, they belong to a ratio scale. But real life problems can involve subtracting numbers from other numbers and the result may be negative numbers as we have done some time ago in making paired comparisons with differences and in scenario construction in planning and with positive and negative priorities as they relate to benefits and costs [2–4]. In this paper, we provide a constructive procedure to extend the idea of priority from positive to negative numbers. What negative numbers mean and how one can derive negative numbers from the beginning is our basic concern. We are solely concerned with negative ratio scales and not with interval scales, so far not explicitly considered in the scientific literature. It does not take a big leap of imagination to recognize the practical usefulness of ratio scales all of whose numbers are assigned negative values in decision making with ratio scales.

It was very early in the development of the AHP/ANP that people lumped together positive and negative aspects of a problem. The simplest and still legitimate use was when people decide to buy a car and cost is in dollars and low cost is committed as a benefit and the reciprocal is used in the same structure. However, it was recognized early that pleasure and pain or gain and loss are not directly comparable. The pain produced by a pin prick to a sensitive part of the body is not a low form of pleasure. It is infinitely better to stroke the sensitive part with a fur than to injure it slightly with the pin.

Negative priorities can be derived from positive dominance comparisons and from ratings just as positive priorities are, except that the sense in which the question is asked in making the comparisons is opposite to that used to derive positive numbers. For example, to derive a positive scale, we ask which of two elements is larger in size or more beautiful in appearance. To derive negative priorities, we ask which of two elements is more costly, or which of two offenses is a worse violation of the law. In a decision, one may have a criterion in terms of which alternatives are found to contribute to a goal in a way that increases satisfaction, and other alternatives contribute in a way that diminishes satisfaction. Here there is symmetry between positive and negative attributes. Some flowers have a pleasant fragrance and are satisfying, whereas other flowers have an unpleasant smell and are dissatisfying; hence, a need for negative numbers to distinguish between the two types of contribution. When several criteria are involved, an alternative may have positive priorities for some as in benefits and opportunities and negative priorities for others as in costs and risks.

Because they are opposite in value to positive priorities, we need a special way to combine the two. Negative numbers on a Cartesian axis are a result of interpreting negative numbers in an opposite sense to the numbers that fall on the positive side. How we make this interpretation is important. In the AHP, we deal with normalized or relative numbers that fall between zero and one. They behave somewhat like probabilities. In practice, probabilities are obtained through counting frequencies of occurrence. In the AHP, the numbers are priorities that are obtained by paired comparisons. In passing, we note that one can also derive probabilities from paired comparisons in response to the question: "Of a pair of events, which is more likely to occur". Thus, it appears that the AHP derives more general scales than those used in probability theory.

Although one does not speak of negative probability, even as one may subtract a probability value from another as in subtracting probabilities from one, often one needs to use negative priorities. While it is true that in ranking a set of objects, first, second, third, and so on, at first glance, negative priorities do not seem to contribute much to this idea of rank, positive and negative numbers together give us a cardinal basis for providing such ordinal ranking. To maintain a tally of positive and negative properties in order to derive priorities, we need negative numbers to obtain the net advantages of the benefits and the costs, the pros and cons of a decision.

In their paper on the performance of the AHP in comparison of gains and losses, Korhonen and Topdagi [5], who were not concerned with the use of negative numbers but only with "when the utility of the objects cannot be evaluated on the same ratio scale", conclude that "the AHP was able surprisingly well to estimate the reasonable utility values for objects. The origin separating utility and disutility scales was estimated as well".

We note that numbers that belong to a ratio scale must, by definition, be always positive. Negative numbers belong to an absolute scale invariant under the identity transformation. Relative measurements are derived as ratio scales and then transformed to absolute numbers like probabilities on an absolute scale. It makes no sense to speak of negative ratio scales. At first look, positive and negative priorities cannot be combined without a unit that makes it possible to relate them. In this paper, we show how to relate positive and negative priorities using the rating protocol of the AHP. One way is to first determine the priorities of the alternatives according to that which is most costly. These priorities are normalized to one. They are then subtracted from one to obtain priorities according to that which is the least costly. In that case, they can be weighted and added to the priorities of the benefits. More generally, when we deal with problems involving both benefits and costs, we can subtract the priorities of the costs from those of the benefits sometimes obtaining negative numbers. But we need to be careful because priorities are relative numbers and it is not legitimate to simply add or subtract the elements of two sets of relative numbers, they first need to be made commensurate. How?

Relative measurement is a theory for trading off different measurements on an absolute scale. In relative measurement, there can be no "absolutes". We may in a conventional way speak of an absolute scale but strictly speaking, it is relative.

Relative measurement in the AHP is derived measurement. Its zero is not absolute but relative to the goal of the specific decision. If one compares stars according to size, their zero is different than the zero of comparing atoms according to size. The zero used to measure atoms and stars on a physical scale is the same absolute zero. An absolute zero requires a unit of measurement. A relative zero does not. To make a relative zero a more general kind, we need to compare atoms with stars according to size so their two relative zeros are combined like all the other numbers into a new zero.

Because we have become accustomed to deal with negative numbers in ordinary arithmetic in a casual way, we tend to assume that they are easy and natural to deal with. But they are not, and the history of negative numbers is a testimony to that. What we will encounter later in the paper justifies the kind of difficulty people had with accepting negative numbers in their thinking even in modern times. Negative priorities present us with their kind of problems that so far have made it difficult for us to accept them as natural in our thinking. To help us come to terms with negative priorities, let us give a brief history of negative numbers and some people's reactions to them.

2. A CURSORY LOOK AT THE HISTORY OF NEGATIVE NUMBERS

The negative of a number compared with that number, is a particular number that when added to the given number equals to zero. The introduction of negative numbers was brought about out of necessity by the development of algebra as the science providing general methods for the solution of arithmetic problems, regardless of content or given numerical data. The need for negative numbers in algebra arose in the solution of problems that reduce to linear equations with one unknown. A possible negative answer in problems of this kind may be interpreted in several ways, for example: oppositely directed segments, motion in a direction opposite to a chosen one, and debt [6].

Unless negative numbers are used, the extensive practice of algebraic methods in solving problems would be extremely difficult. Although it may have been suggested in an older practice by Babylonian astronomers, historians believe that the Hindus were entirely original in creating an idea that proved immensely important later on. They introduced negative numbers to represent debts, and positive numbers to represent assets. Corresponding to each number such as 5, they introduced a new number-5 and called the old numbers positive to distinguish them from the new, negative ones. The Hindus showed, too, that these new numbers could be as useful as positive numbers by employing them to represent debts. In fact, they formulated the arithmetic operations on negative numbers with this application in mind. The idea that zero represents a number, just as any other digit does, is a modern one that was not familiar to medieval thought. The first known use of negative numbers is attributed to Brahmagupta about 628 A.D.; he also states the rules for the four operations with negative numbers. Indian mathematicians used negative roots of equations systematically as early as the sixth to eleventh centuries in problem solving and were interpreted basically as they are today [7–9]. Bhaskara pointed out that the square root of a positive number is twofold, positive and negative. He brings up the matter of the square root of a negative number but says that there is no square root because a negative number is not a square [10]. Though negative numbers had become known in Europe through the Arab texts, most mathematicians of the sixteenth and seventeenth centuries still did not accept them as legitimate, or if they did, would not accept them as roots of equations.

From the Babylonians until the sixteenth century, equations had been stated in the vernacular. Chuquet was the first (1484 A.D.) to write an isolated negative number in an algebraic equation which in modern terminology amounts to $4x = -2$, see [10]. Most authors then felt it necessary to dwell at length on the rules governing multiplication of negative numbers, and some rejected categorically the possibility of multiplication of two negative numbers [11]. Certain aspects of negative numbers were not really well understood until modern times.

It was Descartes (1569–1650) who completely established the use of negative numbers in European science by geometrically interpreting negative numbers as directed line segments. Eliminating the distinction between the positive and negative roots of an equation, in his analytic geometry, the roots of an equation were considered as the coordinates of the points of intersection of a curve with the axis of the independence variable, a triumph for algebra [10]. But doubts about the meaning of negative numbers lingered on. The great mathematician Euler (1707–1783), in the latter half of the eighteenth century, believed that negative numbers were greater than infinity [10]. We believe that we understand Euler's dilemma well in attempting to assign a place to negative numbers in one's thinking even today. The first author has often said that one cannot compare pleasure with pain directly because no matter how small a pain may be, pleasure is infinitely better. In the nineteenth century, Hamilton and others introduced new and abstract ideas (quaternions) that involved the use of negative numbers naturally as part of wider systems of numbers.

The first author had used negative priorities in the context of difference paired comparisons instead of ratios at a six-week symposium on Modules in Applied Mathematics organized by the Mathematical Association of America at Cornell University in the summer of 1976 published in reference [2, pp. 249–250]. Soon after negative priorities were used in scenario construction in planning [3]. One quickly realizes that it is not the idea of using negative numbers for priorities that is intellectually challenging but the creativity needed to combine positive and negative priority scales as we show later in this paper.

It appears to us that there have been four ways in which negative numbers were used:

(1) to solve algebraic equations,
(2) to indicate opposite direction,
(3) to represent points, vectors, and more generally quaternions and octonions, and finally,
(4) in real life to deal arithmetically with credits and debits.

With priorities, which are relative numbers, we may have a different, perhaps a fifth use of negative numbers. We remind the reader that there need not exist a "positive" priority number in a particular decision to which a mirror image negative priority can be added to yield a zero value. For example, there is nothing "below" absolute zero to put negative temperature.

3. GENERAL OBSERVATIONS

The AHP is a multicriteria measurement theory that deals with both tangible and intangible criteria. Before measurement scales were invented, all criteria were intangible and could be dealt with according to what is said about intangibles below. In addition, there are numerous people and situations where utility depends on subjective preferences with little or no regard to what the measurement values say. For example, a rich person may justifiably not have the regard for money that a poor person has, or a pilot have the regard for distances that a camel driver has. This attitude has its own legitimacy and need not be condemned as if it is always in error. It is one important reason why often, software developers have taken an easier route by not paying close attention to the dicta of producing the exact arithmetic results some of us would like to see included in the process of synthesizing priorities. The theory of the AHP in fact has gone far beyond this way for processing intangibles.

When tangibles are present alongside intangibles, they need to be regarded from a theoretical standpoint as if a dogmatic bookkeeper is handling them literally as they are without interjecting subjective interpretation into the process of synthesis. As we shall see later, they are dealt with first in a way to combine the measurements of the alternatives for all tangible criteria using the same ratio scale of measurement into a single overall tangible criterion for that scale. There may be several overall tangible criteria with different scales. They are then prioritized along with the intangible criteria through paired comparisons. Because, for example, tangibles can be added and subtracted, it becomes possible to do the same with normalized values of the alternatives after appropriate prioritization of the criteria.

This prioritization involves assigning a criterion the sum of the measurements of the alternatives with respect to it to the sum of their measurements with respect to all the criteria. The resulting priorities are used to weight the normalized values of the corresponding measurements of the alternatives and the sum is taken for each alternative producing priorities under a single overall positive criterion. The distributive mode, never the ideal mode, is used to combine tangibles with the same scale of measurement. The relative values obtained in this way correspond to the normalized outcomes of the final values of the alternatives obtained from a ratio scale.

If some criteria are negative, all the positive criteria measurements of the alternatives are first combined into a single set of measurement under one criterion by weighting and adding and all those under negative criteria are combined into a single negative criterion also by weighting and adding. The resulting two positive and negative criteria are then assigned weights that correspond to the ratio of the sum of the synthesized priorities of the alternatives under each to the absolute value of the difference between the sums of the values of the alternatives with respect to each of these two (positive and negative) overall criteria.

The overall outcome for the alternatives is obtained by weighting and subtracting their weights with respect to the negative criterion from those with respect to the positive criterion. If the difference of the two sums is zero, one simply uses the sums of the values of the alternatives without dividing by the difference that would in the end cancel when the values are transformed to the ideal mode. The outcome would be proportional to what is obtained by absolute measurement and in the end the ranking of the alternatives is the same. What is of interest now is where to put this composite tangible criterion in the benefits opportunities costs and risks structure ($BOCR$) of intangible criteria discussed in the next two sections. For that structure, the ideal mode is used for all the criteria. The ideal alternative with respect to each criterion is obtained by dividing by the largest value of an alternative. If some of the measurements are negative, one divides by the value of that alternative with the largest absolute value. If the ideal has a value of 1, that tangible composite criterion (say economic) is added to the benefits (B) component of the $BOCR$ structure along with other intangible benefits and weighted by comparing it with the benefits criteria. Negative values of the alternatives after weighting would automatically be added (net effect subtracted from the total). If the ideal value is -1, the alternative's idealized values are multiplied by -1 to invert the signs, and the criterion is added to the costs (C) component of the $BOCR$ structure along other intangible costs and weighted by comparing it with the costs criteria. Negative values are subtracted, and hence, have a positive contribution as they should in the formula $bB + oO - cC - rR$, where B, O, C, R refer to the ideal priorities of the alternatives, and b, o, c, and r refer to the normalized priorities of B, O, C, and R pertaining to the ideal, obtained through rating.

4. RELATIVE NUMBERS AND PRIORITIES [1]

In the real number system, any two numbers can be related by using a unit of measurement so that every number is known as a multiple of that unit and all numbers are related through that unit. The entire number system can be derived in a step-by-step fashion from this very basic concept of a unit. That is what one does with Peano's axioms, the first of which postulates that 1 (the unit) is a natural number, followed by three axioms using the notion of the successor of a number and a fifth one about mathematical induction,

Although there are several other variants (e.g., division be a constant), we are concerned here with a particular kind of relative scale. For us a *relative scale* is a set of numbers r_i, $i = 1 \dots n$, whose ratios, r_i/r_j, are invariant under division by the sum $\sum_{i=1}^{n} r_i$ of all the numbers in the set. It has the following properties:

(1) $0 \leq r_i \leq 1$; and
(2) r_i/r_j; is an absolute number (belongs to a scale that is invariant under the identity transformation).

Priorities are relative numbers derived from paired comparisons according to dominance. Dominance is used to make the paired comparisons. Given a pair of elements and a property or criterion they have in common, one selects the smaller of the two elements as the unit and asks by how many multiples of that unit the larger one dominates the smaller. From all the paired comparisons arranged in a square dominance matrix, one derives the principal eigenvector, a necessary condition, to represent the priorities of the elements. This vector is not unique but is known to belong to a ratio scale (invariant under multiplication by a positive constant—a similarity transformation). It is then transformed to a relative scale by normalization to make it unique or is divided by the value of one of its members (e.g., the largest one) to create an ideal element with unit priority. These are two ways in which priorities are used in the AHP.

Two sets of relative numbers and in particular two sets of priorities without a common element cannot be simply combined into a single set of priorities just as two sets of probabilities of different sort of events cannot be thrown together. They need to be commensurate, which means that some element in one of the two sets must be comparable with an element in the other. When elements are measured directly on two or more criteria having the same scale or unit of measurement, each criterion is assigned a relative weight equal to the sum of the measurements of the elements under it to the total under the other criteria that have the same unit of measurement. Consider choosing the most preferred of three houses with respect to two criteria: price and remodeling cost. The actual dollar values are shown in Table 1. The actual total cost for each house is obtained by simply adding the two numbers; the relative costs are obtained by normalizing these numbers.

Because it also deals with intangibles, the AHP assumes that we do not have the actual dollar values of the houses for the two criteria but only their relative values. In that case, adding these values does not give the same total relative outcome. The two criteria are assigned appropriate relative priorities that are then used to weight the corresponding normalized priorities of the alternatives and add to obtain the overall relative outcome. What numbers should the criteria be assigned that reflect their relative importance to be used in the weighting process? Each criterion must be assigned the relative value of the sum of the values of the alternatives with respect to it, to the total for both. If we do this as shown in Table 2, and multiply and add as in the

Table 1. Unnormalized criteria and alternative weights from measurements in same scale.

Alternatives	Criterion C_1 Unnormalized Weight = 1.0	Criterion C_2 Unnormalized Weight = 1.0	Weighted Sum Unnormalized	Normalized or Relative Values
A_1	200	150	350	350/1300 = .269
A_2	300	50	350	350/1300 = .269
A_3	500	100	600	600/1300 = .462
Column totals	1000	300	1300	1

Table 2. Normalized criteria weights and normalized alternative weights from measurements in same scale (additive synthesis).

Alternatives	Criterion C_1 Normalized Weight = 1000/1300 = 0.7692	Criterion C_2 Normalized Weight = 300/1300 = 0.2308	Weighted Sum
A_1	200/1000	150/300	350/1300 = .269
A_2	300/1000	50/300	350/1300 = .269
A_3	500/1000	100/300	600/1300 = .462

additive synthesis of the AHP, we obtain the right answer. We see that when the alternatives are normalized to obtain relative values, the criteria need to be assigned relative values as above. If the priorities of the alternatives are not normalized, one does not get meaningful answers. The distributive mode of the AHP (obtained by normalizing the values of the alternatives is used rather than the ideal mode (obtained by dividing each value by the largest in the set to establish a unit). It would be meaningless to use the ideal mode in this case because the priorities of the criteria depend on the values of the alternatives. In this manner, for example, one combines all the criteria with dollar measurement into single criterion and all those measured in kilograms into another single criterion in both cases using the distributive mode, and then compares pairwise the resulting two overall criteria as one does other intangible criteria for higher synthesis purposes (whose priorities are now independent of the alternatives) and uses the ideal mode to weight and combine the priorities of the alternatives.

Were we to subtract the values of the alternatives under the second criterion, considered as costs, from those under the first, taken as gains, and then normalize the results, the outcome can again be duplicated in relative terms. We sum each of the two columns for the alternatives under the two criteria and then subtract the second sum from the first. Next, we assign each criterion the sum of the values of the alternatives under it divided by the absolute value of the foregoing difference (treating a difference of zero as a special case in which benefits and costs have equal weights). Note that the criteria are normalized with respect to the difference rather than the sum. We then weight by the priorities of the criteria and add. The outcome, as in the example in Table 3, is identical to subtracting the second value for each alternative from the first summing and then dividing by the absolute value of the sum of these differences. Again the process requires that we use the distributive and not the ideal mode.

In the absence of a unit, at first it is not obvious as to how to combine positive and negative numbers as priorities. Thus, it is not possible, for example, to compare the relative importance of benefits with the relative importance of costs. Pleasure is not a higher form of pain nor is good a higher form of evil. The good and the bad are different but they are opposites of the same kind or dimension. One way that has been justifiably used in the AHP instead of negative priorities is to take reciprocals, weight and then add them to other positive priorities.

We know that people can trade off the benefits of an alternative against its costs in making a decision but they do not do it by a process of wholesale comparison. They do it with respect

Table 3. Criteria weights normalized for subtraction and normalized alternative weights from measurements in the same scale.

Alternatives	Criterion C_1 Normalized Weight = 10/4	Criterion C_2 Normalized Weight = 14/4	Weighted Difference
A_1	3/10	8/14	−5/4
A_2	2/10	1/14	1/4
A_3	5/10	5/14	0

to their own satisfaction (strategic) criteria and by rating the contribution of benefits and the costs of that alternative separately to the fulfillment of those criteria. When there are several alternatives, one uses for each of the benefits and costs that alternative with the largest composite ideal priority. It may be that the best-ranked alternative under benefits differs from that under costs, but in any case, one uses the ideal alternative in developing the weights for the benefits. The same approach applies to the costs. The results obtained from the rating take the form of nonnormalized priorities for the benefits and the costs. Normalizing these rating values yields the desired priorities that enable us to tradeoff the benefits and costs of all the alternatives. The example in the next section will help clarify these ideas.

More generally, in many decision problems, four kinds of concerns are considered: benefits, opportunities, costs, and risks; which we abbreviate as $BOCR$. The first two are advantageous, and hence, are positive and the second two are disadvantageous and are therefore negative. We have sometimes justifiably kept the last two positive in a situation where the decision was already made, for example, to buy a car, and low cost as determined by the normalized reciprocal costs of the alternatives was seen as a benefit that is then weighted and added to the benefits.

An alternative and more accurate way to deal with $BOCR$ is to realize that through normalization of the principal eigenvector, one obtains a dimensionless set of numbers that belong to an absolute scale, invariant under the identity transformation. It is known that absolute numbers can be both positive and negative, and hence, it is not necessary to confine the $BOCR$ to being positive. From the requirement of dominance by using a unit in paired comparisons, we know that we can only ask how much an element dominates another element and not how much an element is dominated by another element. It is not meaningful to do it the opposite way without first using the smaller element as the unit to determine how many times larger is the more dominant element and then estimating the smaller one as a fraction of it. Thus, not only does one ask how much more important one element is than another according to benefits and opportunities, but also how much more costly or risky one element is than another with respect to a certain criterion.

There are at least four ways to combine $BOCR$ priorities with corresponding normalized weights b, o, c, r obtained by rating B, and then C, and then O, and finally, R separately. The first is the traditional one in which weighting amounts to multiplying by the same constant. They are as follows:

(1) BO/CR;
(2) $bB + oO + c(1/C) + r(1/R)$;
(3) $bB + oO + c(1 - C) + r(1 - R)$;
(4) $bB + oO - cC - rR$.

The question now is how to interpret these priorities and use them appropriately in different situations. The first is a tradeoff between a unit of BO against a unit of CR, a unit of the desirable against a unit of the undesirable. The second is a sum of the advantages obtained when committed to action with low values of the disadvantages (the lesser of the evils) considered as good or positive. The third is more optimistic and considers the residual or complementary value, the fact that "not all is bad" as a positive measure. The fourth and last simply subtracts the sum of the weighted "bads" from the sum of the weighted "goods" and can give rise to negative priorities. Either of the first and last ways is a legitimate objective. The last one is total and the first is marginal. If one has infinite resources, there is no question that one would make the necessary investment to get the larger return. But most often the case is that one attempts to maximize return on one's investment. Governments typically take the total approach with their unlimited tax base. But businesses go for the marginal "bang per buck".

When normalized values for the alternatives are used throughout, in the case of reciprocals, they are normalized again after taking the reciprocal. In the idealized case used when the control criteria are independent of the alternatives, although the ideal is used for each control criterion, the synthesized values for each of B, O, C, and R may not have an ideal alternative. We used the idealized reciprocals of the synthesized costs and the risks outcomes for the alternatives and then multiplied these values by what was the largest alternative value before taking the reciprocals in order to rescale the values down from the ideal as they were before taking the reciprocals (should that be the case). Below we give an example and obtain the outcome for these four ways of aggregating $BOCR$.

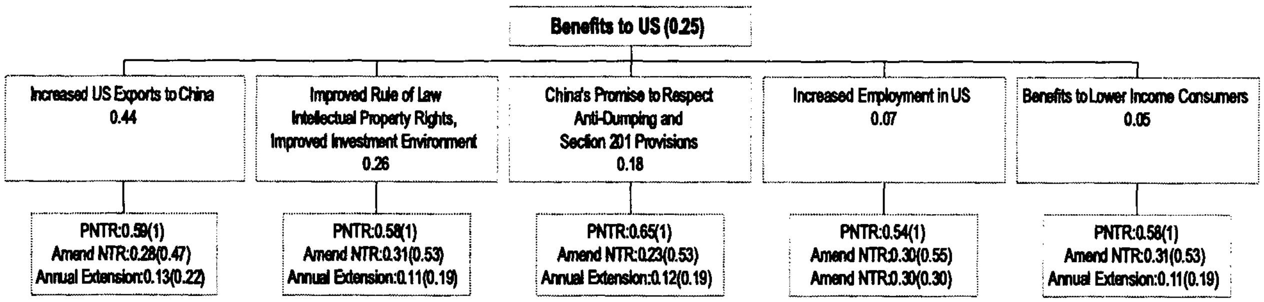
5. EVALUATING THE BOCR MERITS THROUGH STRATEGIC CRITERIA—A RATINGS EXAMPLE [12]

Since 1986, China had been attempting to join the multilateral trade system, the General Agreement on Tariffs and Trade (GATT) and, its successor, the World Trade Organization (WTO)]. According to the rules of the 135-member nation WTO, a candidate member must reach a trade agreement with any existing member country that wishes to trade with it. By the time this analysis was done, China signed bilateral agreements with 30 countries—including the U.S. (November 1999)—out of 37 members that had requested a trade deal with it.

As part of its negotiation deal with the U.S., China asked the U.S. to remove its annual review of China's normal trade relations (NTR) status, until 1998 called most favored nation (MFN) status. In March 2000, President Clinton sent a bill to Congress requesting a permanent normal trade relations (PNTR) status for China. The analysis was done and copies sent to leaders and some members in both houses of Congress before the House of Representatives voted on the bill, May 24, 2000. The decision by the U.S. Congress on China's trade-relations status will have an influence on U.S. interests, in both direct and indirect ways. Direct impacts will include changes in economic, security, and political relations between the two countries as the trade deal is actualized. Indirect impacts will occur when China becomes a WTO member and adheres to WTO rules and principles. China has said that it would join the WTO only if the U.S. gives it permanent normal trade relations status.

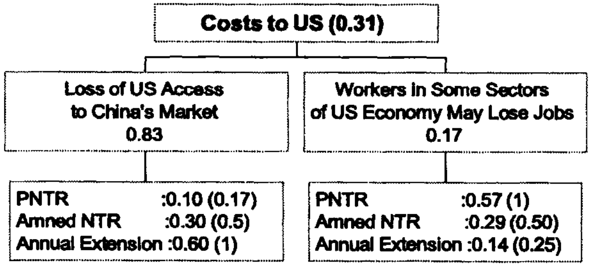
It is likely that Congress will consider four options, the least likely being that the U.S. will deny China both PNTR and annual extension of NTR status. The other three options are as follows.

- *Passage of a clean PNTR bill.* Congress grants China permanent normal trade relations status with no conditions attached. This option would allow implementation of the November 1999 WTO trade deal between China and the Clinton administration. China would also carry out other WTO principles and trade conditions.
- *Amendment of the current NTR status bill.* This option would give China the same trade position as other countries and disassociate trade from other issues. As a supplement, a separate bill may be enacted to address other matters, such as human rights, labor rights, and environmental issues.
- *Annual Extension of NTR status.* Congress extends China's normal trade relations status for one more year, and thus, maintains the status quo.



Benefits Synthesis (Ideal): PNTR: 1 Amend NTR: 0.48 Annual Extension: 0.21

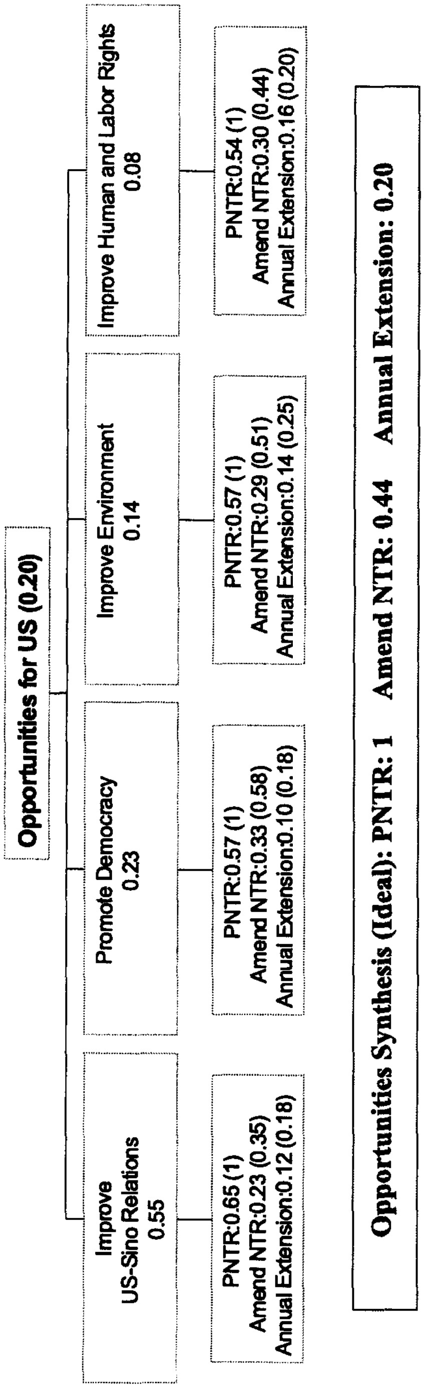
(a)



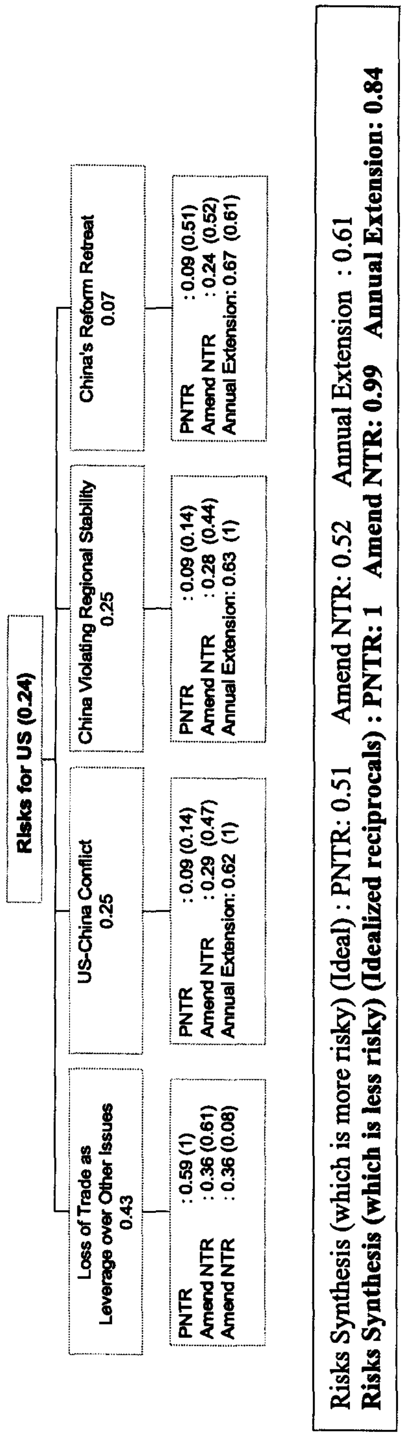
Costs Synthesis (which is more costly) (ideal) :PNTR: 0.31 Amend NTR : 0.50 Annual Extension : 0.87
Costs Synthesis (which is less costly) (Idealized reciprocals) :PNTR: 1 Amend NTR: 0.61 Annual Extension: 0.35

(b)

Figure 1. The four hierarchies for benefits, costs, opportunities, and risks



(c)



(d)

Figure 1. (cont.)

This study was done prior to granting China PNTR status. China now has that status.

Our analysis involves four steps. First, we prioritize the criteria in each of the benefits, costs, opportunities, and risks hierarchies. Figure 1 shows the resulting prioritization of these criteria. The alternatives and their priorities are shown under each criterion both in the distributive and also in the ideal modes. The ideal priorities of the alternatives were used as appropriate to synthesize their final values beneath each hierarchy.

The priorities shown in Figure 1 were derived from judgments that compared the elements involved in pairs. The judgment of preference of one element over another expresses the strength of that preference. This strength can be represented numerically. For readers to estimate the original pairwise judgments (not shown here), one forms the ratio of the corresponding two priorities shown and then takes the closest whole number, or its reciprocal if it is less than 1.0.

It is likely that, in a particular decision, the benefits, costs, opportunities, and risks ($BOCR$) are not equally important, so we must also prioritize them. This is shown in Table 1. The priorities for the economic, security, and political factors themselves were established as shown in Figure 2 and Table 4 and used to rate the importance of the benefits, costs, opportunities, and risks. Finally, we used the priorities of the latter to combine the synthesized priorities of the alternatives in the four hierarchies, using the normalized reciprocal—priorities of the alternatives under costs and risks, to obtain their final ranking, as shown in Table 5.

How to derive the priority shown next to the goal of each of the four hierarchies in Figure 1 is outlined in Table 1. We rated each of the four merits: benefits, costs, opportunities, and risks of the dominant PNTR alternative, in terms of intensities for each assessment criterion. These intensities were prioritized in a matrix as to how much each is preferred over each of the other intensities. We then assigned the appropriate intensity for each merit on all assessment criteria. The outcome was as found in Table 3.

We are now able to obtain the overall priorities of the three major decision alternatives listed earlier, given as columns in Table 4 that gives four ways of synthesis using the ideal mode.

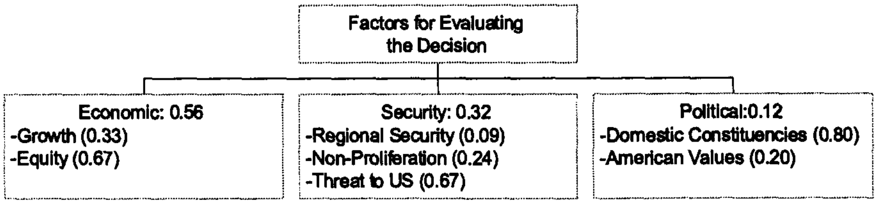


Figure 2. Hierarchy for rating benefits, costs, opportunities, and risks.

Table 4. Priority ratings for the merits: benefits, costs, opportunities, and risks.

		Benefits	Costs	Opportunities	Risks
Economic (0.56)	Growth (0.19)	High	Very Low	Medium	Very Low
	Equity (0.37)	Medium	High	Low	Low
Security (0.32)	Regional (0.03)	Low	Medium	Medium	High
	Nonproliferation (0.08)	Medium	Medium	High	High
	Threat to U.S. (0.21)	High	Very High	High	Very High
Political (0.12)	Constituencies (0.1)	High	Very High	Medium	High
	American Values (0.02)	Very Low	Low	Low	Medium
Priorities		0.25	0.31	0.20	0.24

Very High (0.42), High (0.26), Medium (0.16), Low (0.1), Very Low (0.06).

Table 5. Four methods of synthesizing BOCR using the ideal mode.

Alternatives	Benefits (0.25)	Opportunities (0.20)	Costs (0.31)	Reciprocals of Costs	Costs (Divided by Largest Reciprocal)	Risks (0.24)
PNTR	1	1	0.31	3.23	1	0.51
Amend NTR	0.48	0.44	0.50	2.00	0.62	0.52
Annual Exten.	0.21	0.20	0.87	1.15	0.36	0.61

Alternatives	Reciprocals of Risks	Risks (Divided by Largest Reciprocals	BO/CR	$bB+oO$ $+c(1/C)$ $+r(1/R)$	$bB+oO$ $+c(1-C)$ $+r(1-R)$	$bB+oO$ $-cC-rR$
PNTR	1.96	1	1	0.87	0.78	0.23
Amend NTR	1.92	0.98	0.13	0.52	0.48	−0.07
Annual Exten.	1.64	0.84	0.01	0.31	0.23	−0.32

We give in bold the marginal and the total synthesized values which, along with the two other methods of synthesis, show that PNTR is the dominant alternative.

6. CONCLUSIONS

We have shown here how negative priorities can be defined as relative numbers and used along with positive priorities. We have indicated that in decision making it is not necessary or possible to parallel each positive or negative priority by creating its opposite value. The main contribution of this paper has been to discuss and illustrate negative priorities and how to synthesize the priorities obtained from benefit, opportunity, cost and risk hierarchies. The real conclusion is that we have given several ways of synthesis that can be applied in different situations not specified here. One can use all or select one method that suits one's understanding best. If the rankings are different with the methods, there is information that needs to be considered. In our various applications of these ideas in and outside the classroom, we have used all four to learn more about differences in the resulting rankings.

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