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A fuzzy MCDM approach for evaluating banking performance based on Balanced Scorecard

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ABSTRACT

The paper proposed a Fuzzy Multiple Criteria Decision Making (FMCDM) approach for banking performance evaluation. Drawing on the four perspectives of a Balanced Scorecard (BSC), this research first summarized the evaluation indexes synthesized from the literature relating to banking performance. Then, for screening these indexes, 23 indexes fit for banking performance evaluation were selected through expert questionnaires. Furthermore, the relative weights of the chosen evaluation indexes were calculated by Fuzzy Analytic Hierarchy Process (FAHP). And the three MCDM analytical tools of SAW, TOPSIS, and VIKOR were respectively adopted to rank the banking performance and improve the gaps with three banks as an empirical example. The analysis results highlight the critical aspects of evaluation criteria as well as the gaps to improve banking performance for achieving aspired/desired level. It shows that the proposed FMCDM evaluation model of banking performance using the BSC framework can be a useful and effective assessment tool.

1. Introduction

Financial liberalization and internationalization have been heavily advocated in Taiwan over the past decade, in response to increased global competition. Due to the government's loosening control over the applications of establishing the medium-and-small business banks, the number of domestic headquarters and branches of financial institutions has increased from 6,127 to 6,365 between the years 2000 and 2005 (Central Bank of the Republic of China, 2005). Financial institutions are densely distributed in Taiwan. Moreover, the financial environment of Taiwan has undergone a drastic change since Taiwan entered the World Trade Organization (WTO). It is very important for Taiwan's bank institutions to have a competitive advantage, because they are all quite homogeneous. Therefore, a fiercely competing financial market with relatively little profit, plus the new withdrawal mechanism regulations for low performance banks has resulted in a limited growth of banks in Taiwan. To outperform competing bank institutions, more emphasis on internal operational performance is required. This means it is imperative to develop an effective way to conduct performance evaluations that can measure the overall organizational performance and link it to the corporate goals. That is, a holistic evaluation model of banking performance is key to a bank's survival.

Many different theories and methods of performance for conducting an evaluation have been applied in various organizations for many years. These approaches include ratio analysis, total production analysis, regression analysis, Delphi analysis, Balanced Scorecard, Analytic Hierarchical Process (AHP), Data Envelopment Analysis (DEA) and others. Each method has its own basic concept, aim, advantages and disadvantages (Dessler, 2000). Which one is chosen by management or decision makers for assessing performance depends on the status and type of the organization. However, all the successful enterprises have some common features, including a specific vision, positive actions, and an effective performance evaluation. The Balanced Scorecard (BSC) is an extensive and thorough performance evaluation tool to adequately plan and control an organization so it can attain its goals (Davis & Albright, 2004; Lawrie & Cobbold, 2004; Pinero, 2002). The BSC breaks through the traditional limitations of finance, examining an organization's performance from the four main perspectives of finance, customer, internal business process, and learning and growth (Kaplan & Norton, 1992). It emphasizes both the aspects of the financial and non-financial, long-term and short-term strategies, and emphasizes internal and external business measures. Several studies have been conducted incorporating the four perspectives of the BSC in performance appraisal. To achieve the best possible

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result from a more effective performance, it is crucial to improve the banking relationship by matching the needs of the clients to the delivery process of client services (Nist, 1996). Therefore, the BSC is also utilized as a framework to develop evaluation indicators for banking performance (Davis & Albright, 2004; Kim & Davidson, 2004; Kuo & Chen, 2010).

Since Bellman and Zadeh (1970) developed the theory of decision behavior in a fuzzy environment, various relevant models were developed, and have been applied to different fields such as control engineering, artificial intelligence, management science, and Multiple Criteria Decision Making (MCDM) among others. The concept of combining the fuzzy theory and MCDM is referred to as fuzzy MCDM (FMCDM). Several practicable applications of utilizing FMCDM in criteria evaluation and alternatives selection are demonstrated in previous studies (Bayazita & Karpak, 2007; Chen, Lin, & Huang, 2006; Chiou & Tzeng, 2002; Chiou, Tzeng, & Cheng, 2005; Chiu, Chen, Shyu, & Tzeng, 2006; Hsieh, Lu, & Tzeng, 2004; Lee, Chen, & Chang, 2008; Pepiot, Cheikhrouhou, Furbringer, & Glardon, 2008; Wang & Chang, 2007; Wu & Lee, 2007). Primarily, the MCDM problems are first classified into distinct aspects and different alternatives/strategies and the criteria are defined based on various points of view from stakeholders. Then, a finite set of alternatives/strategies can be evaluated in terms of multi-criteria. Choosing a suitable method to measure the criteria can help the evaluators and analysts to process the cases to be evaluated and determine the best alternative. Like most cases of evaluation, a number of criteria have to be considered for performance appraisal. Consequently, banking performance evaluation can be regarded as a MCDM problem. In addition, the multiple criteria used in the BSC are more objective and comprehensive than a single one. In this research, a FMCDM approach based on the four perspectives of the BSC was proposed to establish a performance evaluation model for bank institutions. The aims of this research are as follows: (1) screen performance indexes to fit the banks for constructing a hierarchical framework of performance evaluation; (2) use FAHP (Fuzzy Analytic Hierarchy Process) to find the fuzzy weights of the indexes by subjective perception; (3) apply SAW (Simple Average Weight), TOPSIS (Technique for Order Preference by Similarity to Ideal Solution method), and VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje) to rank the performance and improve the gaps of three banks in the example; and (4) provide suggestions based on the research results for performance evaluation and serve as a reference for future research in this field.

The remainder of this paper is organized as follows. The concepts of performance evaluation and BSC are introduced and reviewed in Section 2. In Section 3, the performance evaluation framework and the analytical methods used in FMCDM for evaluating the banking performance are proposed. Section 4 provides an empirical example for banking performance, including the hierarchical framework of BSC performance evaluation indexes and the result analyses and discussion to illustrate the proposed performance evaluation model. Section 5 concludes the paper.

2. Performance evaluation and Balanced Scorecard

This section briefly reviews the underlying concepts adopted by this research, such as the definitions of performance evaluation, performance evaluation index, and Balanced Scorecard (BSC).

2.1. Definitions of performance evaluation

The definitions of performance and evaluation are as follows. Performance is referred to as one kind of measurement of the goals of an enterprise, while evaluation is referred to as the goal that an enterprise can effectively obtain during a specific period (Lebas, 1995). Evans, Ashworth, Chellew, Davidson, and Towers (1996) stated that performance evaluation is an important activity of management control, used to investigate whether resources are allocated efficiently; it is applied for the purpose of operational control to achieve a goal adjustment in the short-term and for strategy management and planning in the long run. As indicated by Rue and Byars (2005), performance evaluation tells us how employees define their own work, and it establishes a decision-making and communication process for improvement. Kaplan and Norton (1992) described performance evaluation as a way to review the achievements of organizations of both their financial and non-financial objectives.

There is abundant literature on performance evaluation demonstrating various topics and successful examples relating to performance management (McNamara & Mong, 2005). The traditional performance rankings of banks is based on simple and consistent factors such as financial returns, returns on asset (ROA) and returns on earning (ROE). Nevertheless, performance rankings conducted in this way may not precisely illustrate institutions that embrace strategies for sustaining top performance (Hanley & Suter, 1997). Non-financial criteria such as customer satisfaction, community and employee relations can be vital to a bank's winning strategy, because using only ROA or ROE for performance ranking may not necessarily determine which institution offers the highest returns to the investors, nor does it accurately prove which one is most profitable.

Evaluations of the performance of a bank can be diverse (Kosmidou, Pasiouras, Doumpos, & Zopounidis, 2006). Several previous studies on bank performance measurement examined economies of scale and scope employing traditional statistical methods such as correlation analysis (Arshadi & Lawrence, 1987), translog cost function (Gilligann, Smirlock, & Marshall, 1984; Molyneux, Altunbas, & Gardener, 1996; Murray & White, 1983), loglinear models, or tools like Data Envelopment Analysis (DEA), etc. (Athanassopoulos & Giokas, 2000; Drake, 2001; Giokas, 1991).

2.2. Performance evaluation index

Performance measurement can be defined as a system by which a company monitors its daily operations and evaluates whether the company is attaining its objectives. To fully utilize the function of performance measurement, it is suggested to set up a series of indexes which properly reflect the performance of a company. These indicators can be quantifiable, or unquantifiable. For instance, an index such as lead time is viewed as a quantifiable (or financial) measure, whereas the degree of customer satisfaction is unquantifiable (or non-financial) measures. Managers often have difficulty in delineating strategies and selecting proper measures while implementing the BSC system.

In the early stage of implementing the BSC, it is important to collect as many ideas as possible concerning performance measurement by interviewing business managers and discussing their business vision, mission, and strategies. Meyer and Markiewicz (1997) grouped the measures relating to the critical success factors of banking performance into eights categories: (1) profitability, (2) efficiency and productivity, (3) human resource management, (4) risk management, (5) sales effectiveness, (6) service quality, (7) capital management, and (8) competitive positioning. Collier (1995) employed structural equation models to analyze the process performance of banks using criteria such as process quality errors, employee turnover rate, labor productivity, on-time delivery, and unit cost. The multidimensional indexes used by Arshadi and Lawrence (1987) include profitability, pricing of bank services, and loan market share.

The majority of past studies have focused on customers and how they choose the bank that will offer them general bank services. According to the related literature, the selection criteria which customers use to evaluate and choose between banks, include price, speed, access, customer service, location, image and reputation, modern facilities, interest rates, opening hours, incentive offered, product range, and service charge policy and so on (Anderson, Cox, & Fulcher, 1976; Boyd, Leonard, & White, 1994; Chia & Hoon, 2000; Devlin, 2002; Devlin & Gerrard, 2005; Elliot, Shatto, & Singer, 1996; Martenson, 1985). The more recent research of Devlin and Gerrard (2005) made an attempt to address the relative importance of various choice criteria in the selection of a banking institution by applying a quantitative methodology of statistical analysis. They provided an analysis of customer choice criteria and multiple banking and made an itemized comparison of the relative importance of choice criteria which impact on the choice for main and secondary banking institutions.

2.3. Balanced Scorecard

The concept of Balanced Scorecard (BSC) was proposed by David Norton, the CEO of Nolan Norton Institute, and Robert Kaplan, a professor at Harvard University (Kaplan & Norton, 1992). The BSC measures organizational performance from four perspectives, including financial, customer, internal business process, and learning and growth, in relation to the four functions of accounting and finance, marketing, value chain, and human resource. These measures, both financial and non-financial, from all four perspectives serve as the common language to help align the top management and employees toward with the organization's vision. The BSC provides managers with the instrumentation tools they need to navigate towards future competitive success (Kaplan & Norton, 1992, 1996a, 1996b). The essential tenet of the BSC is that standard financial measures must be balanced with non-financial measures (Norton, Contrada, & LoFrumento, 1997).

There has been generally accepted in practice that since the introduction of the BSC by Kaplan and Norton a combination of financial and non-financial measures in a performance measurement system is favorable for both profit and non-profit organizations (Ballou, Heitger, & Tabor, 2003; Sinclair & Zairi, 2001). Banks can save both time and money if they recognize which measures are most suitable for their needs. Non-financial measures such as intangibles like customer relationships may account for more than half of the total assets of a company. An important principle of the BSC is to achieve success on key non-financial measures before actualizing success on key financial measures. When considered in non-financial measures to other measures, these metrics can lead organizations to administer performance effectively and forecast their future profitability (Anonymous, 2006; Mouritsen, Thorsgaard, & Bukh, 2005).

The BSC is a popular tool that is applied by many businesses to assess their performance in diverse aspects of their organization. It provides insights into corporate performance not only for managers seeking ways to improve performance, but also for investors wanting to gauge the organizations' ongoing health. For banks the benefits of using BSC are numerous: (1) can be used as a framework to assess and develop a bank's strategy; (2) can be used to develop strategic objectives and performance measures to transform a bank's strategy into action; (3) it provides a way to measure and monitor the performance of key performance drivers that may lead to the successful execution of a bank's strategy; and (4) it is an effective tool to ensure that a bank continuously improves its system and process (Frigo, Pustorino, & Krull, 2000). Davis and Albright (2004) presented an empirical analysis that explores the effect of the BSC on a banking institution's financial performance. Kim and Davidson (2004) used the BSC framework to assess the business performance of information technology (IT) expenditures in the banking industry using the *t*-test and regression models. Kuo and Chen (2010) applied the four perspectives of the BSC to construct key performance appraisal indicators for the mobility of the service industries through the fuzzy Delphi method. Leung, Lam, and Cao (2006) proposed a tailor-made performance measurement model using the analytic hierarchy process and the analytic network process for implementing the BSC.

A large amount of research related to the financial industry employed the BSC to evaluate performance and has benefited from its use (Ashton, 1998; Davis & Albright, 2004). Nevertheless, most of these studies focused on how to set up an effective mechanism to select evaluation criteria rather than on calculating their relative weight. Therefore, this research aims at developing an evaluation model for banking performance not only to investigate the relative importance among the selection criteria, but also to examine the critical gaps for achieving aspired/desired level.

3. Performance evaluation framework and analytical methods

The analytical structure of this research is illustrated in Fig. 1. A performance analysis is conducted based on the selected evaluation criteria. First the FAHP approach is employed to calculate the relative weights of the performance evaluation indexes. Then, according to these weights the three MCDM analytical tools of SAW, TOPSIS, and VIKOR are used to rank and improve the banking performance and determine the best practice. The concepts of the fuzzy set theory and details of the analytical methods are explained in the following subsections.

3.1. Fuzzy set theory

Expressions such as "not very clear", "probably so", and "very likely", are used often in daily life, and more or less represent some degree of uncertainty of human thought. The fuzzy set theory proposed by Zadeh (1965), an important concept applied in the scientific environment, has been available to other fields as well. Consequently, the fuzzy theory has become a useful tool for automating human activities with uncertainty-based information. Therefore, this research incorporates the fuzzy theory into the performance measurement by objectifying the evaluators' subjective judgments.

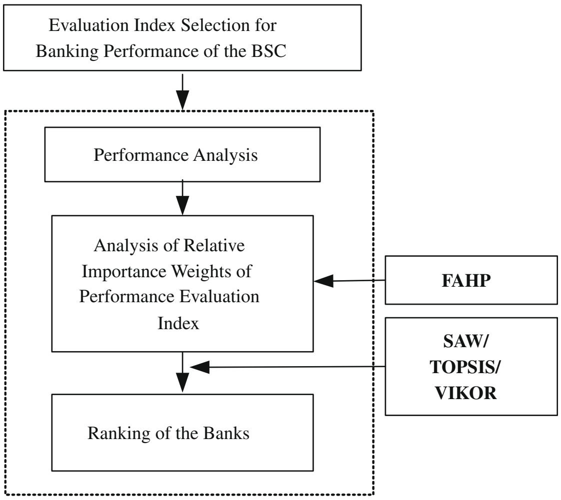

Fig. 1. Performance evaluation framework of the research.

3.1.1. Fuzzy number

In the classical set theory, the truth value of a statement can be given by the membership function as $\mu_A(x)$

$$\mu_A\ x \quad \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases} \qquad 1$$

Fuzzy numbers are a fuzzy subset of real numbers, and they represent the expansion of the idea of a confidence interval. According to the definition by Dubois and Prade (1978), the fuzzy number $\widetilde{A}$ is of a fuzzy set, and its membership function is $\mu_{\widetilde{A}}\ x\ : R \rightarrow\ 0,1 \times 0 \leqq \mu_{\widetilde{A}}\ x\ \leqq 1, x \in X$, where x represents the criterion and is described by the following characteristics: (1) $\mu_{\widetilde{A}}\ x$ is a continuous mapping from R (real line) to the closed interval [0,1]; (2) $\mu_{\widetilde{A}}\ x$ is of a convex fuzzy subset; (3) $\mu_{\widetilde{A}}\ x$ is the normalization of a fuzzy subset, which means that there exists a number x_0 such that $\mu_{\widetilde{A}}\ x0 \quad 1$. For instance, the triangular fuzzy number (TFN), $\widetilde{A} \quad l,m,u$, can be defined as Eq. (2) and the TFN membership function is shown in Fig. 2:

$$\mu_{\widetilde{A}}\ x \quad \begin{cases} x * l\ /\ m * l & \text{if } l \leqslant x \leqslant m, \\ u * x\ /\ u * m & \text{if } m \leqslant x \leqslant u, \\ 0 & \text{otherwise}. \end{cases} \qquad 2$$

Based on the characteristics of TFN and the extension definitions proposed by Zadeh (1975), given any two positive triangular fuzzy numbers, $\widetilde{A}_1 \quad l_1, m_1, u_1$ and $\widetilde{A}_2 \quad l_2, m_2, u_2$, and a positive real number r, some algebraic operations of the triangular fuzzy numbers $\widetilde{A}_1$ and $\widetilde{A}_2$ can be expressed as follows:

Addition of two TFNs :

$$\widetilde{A}_1 \quad \widetilde{A}_2 \quad l_1 \quad l_2, m_1 \quad m_2, u_1 \quad u_2\ . \qquad 3$$

Multiplication of two TFNs :

$$\widetilde{A}_1 \quad \widetilde{A}_2 \quad l_1 l_2, m_1 m_2, u_1 u_2\ . \qquad 4$$

Multiplication of any real number r and a TFN :

$$r \quad \widetilde{A}_1 \quad rl_1, rm_1, ru_1 \quad \text{for } r > 0 \quad \text{and} \quad l_i > 0,\ m_i > 0,\ u_i > 0 \qquad 5$$

Subtraction of two TFNs Θ:

$$\widetilde{A}_1\ \Theta\ \widetilde{A}_2 \quad l_1 * u_2, m_1 * m_2, u_1 * l_2 \quad \text{for } l_i > 0,\ m_i > 0,\ u_i > 0. \qquad 6$$

Division of two TFNs $\varnothing$:

$$A_1 \varnothing A_2 \quad l_1/u_2, m_1/m_2, u_1/l_2\ . \qquad 7$$

Reciprocal of a TFN:

$$\widetilde{A}_1^{*1} \quad 1/u_1, 1/m_1, 1/l_1 \quad \text{for } l_i > 0,\ m_i > 0,\ u_i > 0. \qquad 8$$

3.1.2. Linguistic variable

Linguistic variables are variables whose values are words or sentences in a natural or artificial language. In other words, they are variables with lingual expression as their values (Hsieh et al., 2004; Zadeh, 1975). The possible values for these variables could be: "very dissatisfied", "not satisfied", "fair", "satisfied", and "very satisfied". The evaluators are asked to conduct their judgments, and each linguistic variable can be indicated by a triangular fuzzy number (TFN) within the scale range of 0–100. An example of membership functions of five levels of linguistic variables is shown in Fig. 3. For instance, the linguistic variable "Satisfied" can be represented as (60, 80, 100). Besides, each evaluator can personally define his/her subjective range of linguistic variables. The use of linguistic variables is applied widely. In this paper, linguistic variables expressed by TFN are adopted to stand for evaluators' subjective measures to determine the degrees of importance among evaluation criteria and also assess the performance value of alternatives.

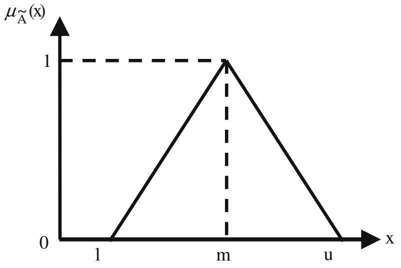

Fig. 2. Membership function of the triangular fuzzy number.

3.2. Fuzzy analytic hierarchy process

The Analytic Hierarchy Process (AHP) was devised by Saaty (1980, 1994). It is a useful approach to solve complex decision problems. It prioritizes the relative importance of a list of criteria (critical factors and sub-factors) through pairwise comparisons amongst the factors by relevant experts using a nine-point scale. Buckley (1985) incorporated the fuzzy theory into the AHP, called the Fuzzy Analytic Hierarchy Process (FAHP). It generalizes the calculation of the consistent ratio (CR) into a fuzzy matrix. The procedure of FAHP for determining the evaluation weights are explained as follows:

Step 1: *Construct fuzzy pairwise comparison matrices.* Through expert questionnaires, each expert is asked to assign linguistic terms by TFN (as shown in Table 1 and Fig. 4) to the pairwise comparisons among all criteria in the dimensions of a hierarchy system. The result of the comparisons is constructed as fuzzy pairwise comparison matrices $\widetilde{\boldsymbol{A}}$ as shown in Eq. (9).

Step 2: *Examine the consistency of the fuzzy pairwise comparison matrices.* According to the research of Buckley (1985), it proves that if $\boldsymbol{A} = [a_{ij}]$ is a positive reciprocal matrix then $\widetilde{\boldsymbol{A}} \quad \tilde{a}_{ij} \times$is a fuzzy positive reciprocal matrix. That is, if the result of the comparisons of $\boldsymbol{A} = [a_{ij}]$ is consistent, then it can imply that the result of the comparisons of $\widetilde{\boldsymbol{A}} \quad \tilde{a}_{ij} \times$ is also consistent. Therefore, this research employs this method to validate the questionnaire

$$\widetilde{A} \quad \begin{bmatrix} 1 & \tilde{a}_{12} & & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & \tilde{a}_{12} & & \tilde{a}_{1n} \\ 1/\tilde{a}_{12} & 1 & & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 1/\tilde{a}_{1n} & 1/\tilde{a}_{2n} & & 1 \end{bmatrix}. \qquad 9$$

Step 3: *Compute the fuzzy geometric mean for each criterion.* The geometric technique is used to calculate the geometric mean $\tilde{r}_i$ of the fuzzy comparison values of criterion i to each criterion, as shown in Eq. (10), where $\tilde{a}_{in}$ is a fuzzy value of the pair-wise comparison of criterion i to criterion n (Buckley, 1985)

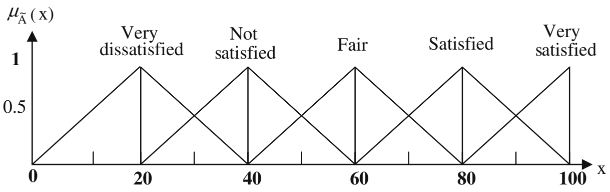

Fig. 3. Membership functions of the five levels of linguistic variables.

Table 1
Membership function of the linguistic scale.

Fuzzy number	Linguistic scales	TFN $\tilde{a}_{ij}$	Reciprocal of a TFN $\tilde{a}_{ij}$
$\tilde{9}$	Absolutely important	(7,9,9)	(1/9,1/9,1/7)
$\tilde{7}$	Very strongly important	(5,7,9)	(1/9,1/7,1/5)
$\tilde{5}$	Essentially important	(3,5,7)	(1/7,1/5,1/3)
$\tilde{3}$	Weakly important	(1,3,5)	(1/5,1/3,1)
$\tilde{1}$	Equally important	(1,1,3)	(1/3,1,1)
$\tilde{2},\tilde{4},\tilde{6},\tilde{8}$	Intermediate value between two adjacent judgments		

Source: Mon et al. (1994) and Hsieh et al. (2004).

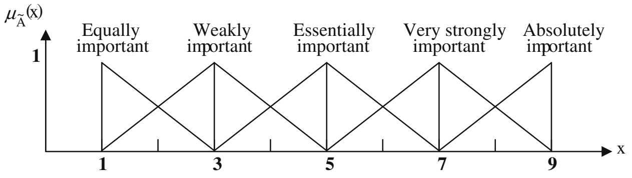

Fig. 4. Membership functions of the linguistics variables for criteria comparisons.

$$\tilde{r}_i \quad \tilde{a}_{i1} \qquad \tilde{a}_{in} \times^{1/n}. \tag{10}$$

Step 4: Compute the fuzzy weights by normalization. The fuzzy weight of the ith criterion $\tilde{\boldsymbol{w}}_i$, can be derived as Eq. (11), where $\tilde{\boldsymbol{w}}_i$ is denoted as $\tilde{\boldsymbol{w}}_i \quad L_{w_i}, M_{w_i}, U_{w_i}$ by a TFN and L_{w_i}, M_{w_i}, and U_{w_i} represent the lower, middle and upper values of the fuzzy weight of the ith criterion

$$\tilde{\boldsymbol{w}}_i \quad \tilde{r}_i \quad \tilde{r}_1 \quad \tilde{r}_2 \qquad \tilde{r}_n \ ^{*1}. \tag{11}$$

3.3. The synthetic value of fuzzy judgment

Since Bellman and Zadeh (1970) proposed the decision-making methods in fuzzy environments, an increasing number of related models have been applied in various fields, including control engineering, expert system, artificial intelligence, management science, operations research, and MCDM. The above decision-making problems were solved by applying the fuzzy set theory. This approach is called the fuzzy MCDM (FMCDM). Its main application is focused on criteria evaluation or project selection. The FMCDM method can assist decision makers in selecting the best alternative, or ranking the order of projects.

Due to the differences in the subjective judgments among the experts for each evaluation criterion, the overall valuation of the fuzzy judgment is employed to synthesize the various experts' opinions in order to achieve a reasonable and objective evaluation. The calculation steps to obtain the synthetic value are:

Step 1: Performance evaluation of the alternatives. As shown in Fig. 3, "very dissatisfied", "not satisfied", "fair", "satisfied", and "very satisfied" are the five linguistic variables used to measure the performance of the alternatives against the evaluation criteria. Each linguistic variable can be presented by a TFN with a range of 0–100. Assume that $\widetilde{E}_{ij}^k$ denotes the fuzzy valuation of performance given by the evaluator k towards alternative i under criterion j as Eq. (12) shows, then:

$$\widetilde{E}_{ij}^k \quad \left(LE_{ij}^k, ME_{ij}^k, UE_{ij}^k\right). \tag{12}$$

In this research, $\widetilde{E}_{ij}$ represents the average fuzzy judgment values integrated by m evaluators as

$$\widetilde{E}_{ij} \quad 1/m \quad \left(\widetilde{E}_{ij}^1 \quad \widetilde{E}_{ij}^2 \qquad \widetilde{E}_{ij}^m\right). \tag{13}$$

According to Buckley (1985), the three end points of $\widetilde{E}_{ij}$ can be computed as

$$LE_{ij} \quad \left(\sum_{k \ 1}^{m} LE_{ij}^k\right)\Big/m, \quad ME_{ij} \quad \left(\sum_{k \ 1}^{m} ME_{ij}^k\right)\Big/m,$$
$$UE_{ij} \quad \left(\sum_{k \ 1}^{m} UK_{ij}^k\right)\Big/m. \tag{14}$$

Step 2: Fuzzy synthetic judgment. According to the fuzzy weight, $\tilde{w}_j$, of each criterion calculated by FAHP, the criteria vector $\tilde{\boldsymbol{w}}$ is derived as Eq. (15). And, the fuzzy performance matrix $\widetilde{\boldsymbol{E}}$, as presented in Eq. (16), of all the alternatives can be acquired from the fuzzy performance value of each alternative under n criteria.

$$\tilde{\boldsymbol{w}} \quad \tilde{w}_1, \ldots, \tilde{w}_j, \ldots, \tilde{w}_n \ ^t, \tag{15}$$
$$\widetilde{\boldsymbol{E}} \quad \tilde{e}_{ij} \times \tag{16}$$

Then, the final fuzzy synthetic decision can be deduced from the criteria weight vector $\tilde{\boldsymbol{w}}$ and the fuzzy performance matrix $\widetilde{\boldsymbol{E}}$; and then the derived result, the final fuzzy synthetic decision matrix $\widetilde{\boldsymbol{R}}$, is calculated by $\widetilde{\boldsymbol{R}} \quad \widetilde{\boldsymbol{E}} \Longleftrightarrow \tilde{\boldsymbol{w}}$, where the sign, $\Leftrightarrow$, indicates the computation of the fuzzy number, consisting of both fuzzy addition and fuzzy multiplication. Considering that the computation of fuzzy multiplication is rather complicated, the approximate multiplied result of the fuzzy multiplication is used here. For instance, the approximate fuzzy number $\widetilde{\boldsymbol{R}}_i$ of the fuzzy synthetic decision of the alternative i is denoted as Eq. (17), where LR_i, MR_i, and UR_i are the lower, middle, and upper synthetic performance values of alternative i, respectively, and the calculations of each are illustrated as Eq. (18)

$$\widetilde{R}_i \quad LR_i, MR_i, UR_i \ , \tag{17}$$

where $LR_i \quad \sum_{j \ 1}^{n} Lw_j \quad LE_{ij}, \quad MR_i \quad \sum_{j \ 1}^{n} Mw_j \quad ME_{ij},$

$$UR_i \quad \sum_{j \ 1}^{n} Uw_j \quad UE_{ij}. \tag{18}$$

Next, the procedure of defuzzification (Hsieh et al., 2004; Opricovic & Tzeng, 2003) locates the Best Nonfuzzy Performance value (BNP). Methods used in such defuzzified fuzzy ranking generally include the mean of maximal (MOM), center of area (COA), and α-cut. Utilizing the COA method to find out the BNP is a simple and practical without the need to bring in the preferences of any evaluators. Therefore it is used in this study. The BNP value of the fuzzy number $\widetilde{R}_i$ can be found by

$$\mathrm{BNP}_i \quad UR_i * LR_i \quad MR_i * LR_i \not{/}3 \quad LR_i \quad \forall i. \tag{19}$$

The ranking of the alternatives then proceeds based on the value of the derived BNP for each of the alternatives.

3.4. TOPSIS method

TOPSIS was developed by Hwang and Yoon (1981), based on the concept that the chosen/improved alternatives should be the shortest distance from the positive ideal solution (PIS) and the farthest from the negative-ideal solution (NIS) for solving a MCDM problem. Thus, the best alternative should not only be the shortest distance away from the positive ideal solution (aspired/desired level), but also should be the largest distance away from the negative ideal solution (tolerable level). In short, the ideal solution is composed of all the criteria with the best values attainable (aspired/desired levels), whereas the negative ideal solution is made up of all the criteria with the worst values attainable (tolerable level). The general step-by-step procedure using the TOPSIS is briefly listed as follows.

Step 1: Establish the original performance matrix. The structure of the performance matrix (X) is shown as Eq. (20), where A_i denotes the alternative i, $i = 1,2,\ldots,m$; C_j is the jth criterion, $j = 1,2,\ldots,n$. Therefore, x_{ij} represents the performance value of alterative i in criterion j

$$\begin{array}{cc} & \begin{array}{ccccc} C_1 & \cdots & C_j & \cdots & C_n \end{array} \\ \mathbf{X} = \begin{array}{c} A_1 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{array} & \left[\begin{array}{ccccc} x_{11} & \cdots & x_{1j} & \cdots & x_{1n} \\ \vdots & & \vdots & & \vdots \\ x_{i1} & \cdots & x_{ij} & \cdots & x_{in} \\ \vdots & & \vdots & & \vdots \\ x_{m1} & \cdots & x_{mj} & \cdots & x_{mn} \end{array}\right] \\ & \begin{array}{cccccl} x_1^* & \cdots & x_j^* & \cdots & x_n^* & \text{Aspired level} \\ x_1^- & \cdots & x_j^- & \cdots & x_n^- & \text{Tolerable/worst level} \end{array} \end{array} \qquad 20$$

Step 2: Calculate the normalized performance matrix. The purpose of normalizing the performance (including: the larger is better and the smaller is better) is to remove the units of matrix entries by converting the performance values to a range between 0 and 1. The normalized value (r_{ij}) is calculated as

$$r_{ij} \quad \frac{|x_{ij} \ast x_j^*|}{|x_j^{\oplus} \ast x_j^*|}, \quad i \quad 1,2,\ldots,m; \quad j \quad 1,2,\ldots,n. \qquad 21$$

Step 3: Compute the weighted normalized performance matrix. Considering that there is a difference in the importance of the criteria, the normalized performance matrix has to be weighted as illustrated in Eq. (22), where w_j is the weight of the criterion j, and v_{ij} is the weighted normalized performance matrix. The summation of w_j is equal to 1

$$v_{ij} \quad w_j \quad r_{ij}. \qquad 22$$

Step 4: Determine the ideal (aspired/desired) and negative-ideal (tolerable/worst) solutions. The ideal and negative ideal solutions (A^* and A^*) are elaborated as Eqs. (23) and (24) respectively, where C_b is associated with the benefit criteria, and C_c is associated with the cost criteria

$$A^{\oplus} \quad \{ \max_i v_{ij} | j \in C_b \;, \; \min_i v_{ij} | j \in C_c \;\} \qquad 23$$

or setting the aspired/desired level

$$x_1^{\oplus},\ldots,x_j^{\oplus},\ldots,x_n^{\oplus} \quad \left\{ v_j^{\oplus} | j \quad 1,2,\ldots,n \right\}$$

or $A^{\oplus} \quad \{w_1,\ldots,w_j,\ldots,w_n\}$,

$$A^* \quad \{ \min_i v_{ij} | j \in C_b \;, \; \max_i v_{ij} | j \in C_c \;\} \qquad 24$$

or setting the tolerable level

$$0,\ldots,0,\ldots,0 \quad \{v_j^* \,| j \quad 1,2,\ldots,n\} \text{ or } A^* \quad 0,\ldots,0,\ldots,0$$

Step 5: Calculate the separation measures. The distance can be calculated by the n-dimensional Euclidean distance. The separations of each alternative from the ideal solution $d_i^{\oplus}$ and the negative-ideal solution d_i^* are defined as Eqs. (25) and (26) respectively.

$$d_i^{\oplus} \quad \sqrt{\sum_{j \; 1}^{n} v_{ij} \ast v_j^{\oplus} \; {}^2} \quad \forall i, \qquad 25$$

$$d_i^* \quad \sqrt{\sum_{j \; 1}^{n} v_{ij} \ast v_j^* \; {}^2} \quad \forall i. \qquad 26$$

Step 6: Calculate the relative closeness (similarity) to the ideal solution and rank the preference order. The relative closeness of alternative A_i with respect to $\boldsymbol{A}^*$ can be expressed as Eq. (27), where the index value of $RC_i^{\oplus}$ is between 0 and 1.

$$RC_i^{\oplus} \quad \frac{d_i^*}{d_i^{\oplus} \quad d_i^*} \quad 1 \ast \frac{d_i^{\oplus}}{d_i^{\oplus} \quad d_i^*}, \qquad 27$$

where $RC_i^{\oplus}$ shows that the larger the index value, the better the performance of the alternatives and

$$\left\{ \frac{d_i^{\oplus}}{d_i^{\oplus} \quad d_i^*} \middle| i \quad 1,2,\ldots,m \right\} \qquad 28$$

Eq. (28) denotes a relative indicator of the synthetic gap in alternative i caused by j criterion and $j = 1,2,\ldots,n$. The synthetic gap is the main issue in this problem. How can we improve/reduce the gaps to reach zero so as to achieve the aspired/desired level in each criterion? The TOPSIS method used to provide the information for improving the gaps in each criterion cannot be used for ranking purpose (see Opricovic & Tzeng, 2004, pp. 450–456 and Fig. 2). Therefore, the authors propose the VIKOR method for ranking and improving the alternatives of this problem. This method is introduced in the Section 3.5.

3.5. VIKOR

The Multi-criteria Optimization and Compromise Solution (called VIKOR) is a suitable tool to evaluate each alternative for each criterion function (Opricovic & Tzeng, 2002, 2003, 2004, 2007; Tzeng, Lin, & Opricovic, 2005). The concept of VIKOR is based on the compromise programming of MCDM by comparing the measure of "closeness" to the "ideal" alternative. The multi-criteria measure for compromise ranking is developed from the L_p-*metric* that is used as an aggregating function in compromise programming (Yu, 1973; Zeleny, 1982). The compromise ranking algorithm of VIKOR consists of the following steps:

Step 1: Determine the be st (aspired/desired levels) and worst values (tolerable/worst levels). Assuming that jth criterion represents a benefit, then the best values for setting all the criteria functions (aspired/desired levels) are $\{x_j^{\oplus} | j \quad 1, 2,\ldots,n\}$ and the worst values (tolerable/worst levels) are $\{x_j^* \,| j \quad 1,2,\ldots,n\}$, respectively.

Step 2: Compute the gaps $\{S_i | i \quad 1,2,\ldots,m\}$ and $\{R_i | i \quad 1,2,\ldots,m\}$ from the L_p-metric referring to Eq. (29) by normalization. The relationships are presented in Eqs. (30) and (31)

$$d_i^p \quad \left\{ \sum_{j \; 1}^{n} \left(w_j \frac{|x_j^{\oplus} \ast x_{ij}|}{|x_j^{\oplus} \ast x_j^* \,|} \right)^p \right\}^{1/p}, \quad i \quad 1,2,\ldots,m, \qquad 29$$

$$S_i \quad d_i^{p \;\; 1} \quad \sum_{j \; 1}^{n} \left(w_j \frac{|x_j^{\oplus} \ast x_{ij}|}{|x_j^{\oplus} \ast x_j^* \,|} \right), \quad i \quad 1,2,\ldots,m, \qquad 30$$

$$R_i \quad d_i^{p \;\; \infty} \quad \max_j \left\{ w_j \frac{|x_j^{\oplus} \ast x_{ij}|}{|x_j^{\oplus} \ast x_j^* \,|} | j \quad 1,2,\ldots,n \right\},$$

$$i \quad 1,2,\ldots,m, \qquad 31$$

where S_i, $R_i \in [0,1]$ and 0 denotes the best (i.e., achieving aspired/desired level) and 1 denotes the worst situations.

Step 3: Compute the gaps $\{Q_i | i \quad 1, 2, \ldots, m\}$ for ranking. The relation is defined as Eq. (32), where $S^{\oplus} \quad \min_i S_i$ (the best S^* can be set equal zero), $S^* \quad \max_i S_i$ (the worst S^* can be set to equal one); $R^{\oplus} \quad \min_i R_i$ (the best R^* can be set to equal zero), $R^* \quad \max_i R_i$ (the worst R^* can be set to equal one), and $v \in [0,1]$ is introduced as the weight of the strategy of the "the majority of the criteria" (or "maximum group utility"), usually $v = 0.5$. In this research, the value of v is set to equal 0, 0.5 and 1 for sensitive analysis.

$$Q_i \quad v\left[\frac{S_i * S^{\oplus}}{S^* * S^{\oplus}}\right] \quad 1 * v\left[\frac{R_i * R^{\oplus}}{R^* * R^{\oplus}}\right], \quad i \quad 1, 2, \ldots, m, \qquad 32$$

Step 4: Rank and improve the alternatives, sort by the values S, R, and Q, in decreasing order and reduce the gaps in the criteria. The results are three ranking lists, with the best alternatives having the lowest value.

Step 5: Propose a compromise solution. For a given criteria weight, the alternatives (a'), are the best ranked by measure Q (minimum) if the following two conditions are satisfied:

C1. "Acceptable advantage": $Q(a'') * Q(a') \geqslant DQ$, where a'' is the alternative with second position in the ranking list by Q; $DQ = 1/(J * 1)$; J is the number of alternatives.

C2. "Acceptable stability in decision making": Alternative a' must also be the best ranked by S or/and R. This compromise solution is stable within a decision making process, which could be: "voting by majority rule" (when $v > 0.5$ is needed), or "by consensus"$v \quad 0.5$, or "with veto" ($v < 0.5$). Here, v is the weight of decision making strategy "majority of criteria" (or "the maximum group utility").

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, consisting of:

⊗ Alternatives a' and a'' if only condition C2 is not satisfied, or
⊗ Alternatives$a', a'', \ldots, a^{(M)}$ if condition C1 is not satisfied; and $a^{(M)}$ is determined by the relation $Q(a'') * Q(a') < DQ$ for maximum M (the positions of these alternatives are "in closeness").

The compromise solution obtained by VIKOR can be accepted by the decision makers because it provides a maximum "group utility" of the "majority" (with measure S, representing "concordance"), and a minimum individual regret of an "opponent" (with measure R, representing "discordance"). The compromise solutions can be the basis for negotiations, by involving the criteria weights of the decision makers' preference.

4. An empirical example for banking performance

The four perspectives of BSC were taken as the framework for establishing performance evaluation indexes in this research. Based on this framework, the FAHP was used to obtain the fuzzy weights of the indexes. The three MCDM analytical tools, SAW, TOPSIS, and VIKOR were respectively applied to evaluate the banking performance based on the weight of each index, and to improve the gaps with three banks as an empirical example. The hierarchical framework of the BSC performance evaluation criteria, and the results, analyses and discussions of the empirical example are illustrated in the following section.

4.1. Hierarchical framework of the BSC performance evaluation criteria

From the four BSC perspectives, and based on a review of the literature, 55 evaluation indexes related to banking performance were summarized. Then, expert questionnaires were used for screening the indexes fit for the banking performance evaluation. Twenty-three evaluation indexes were selected by the committee of experts, comprised of twelve professionals from practice and the academia. The descriptions of the criteria for the selection evaluation of a bank's performance are listed in the appendix. The hierarchical framework of the BSC performance evaluation criteria (*i.e.* four dimensions and 23 indexes) for banking is shown in Fig. 5. The 23 evaluation indexes are grouped into the four BSC dimensions, "F: Finance (F1–F6)", "C: Customer (C1–C6)", "P: Internal Process (P1–P6)", and "L: Learning and Growth (L1–L5)".

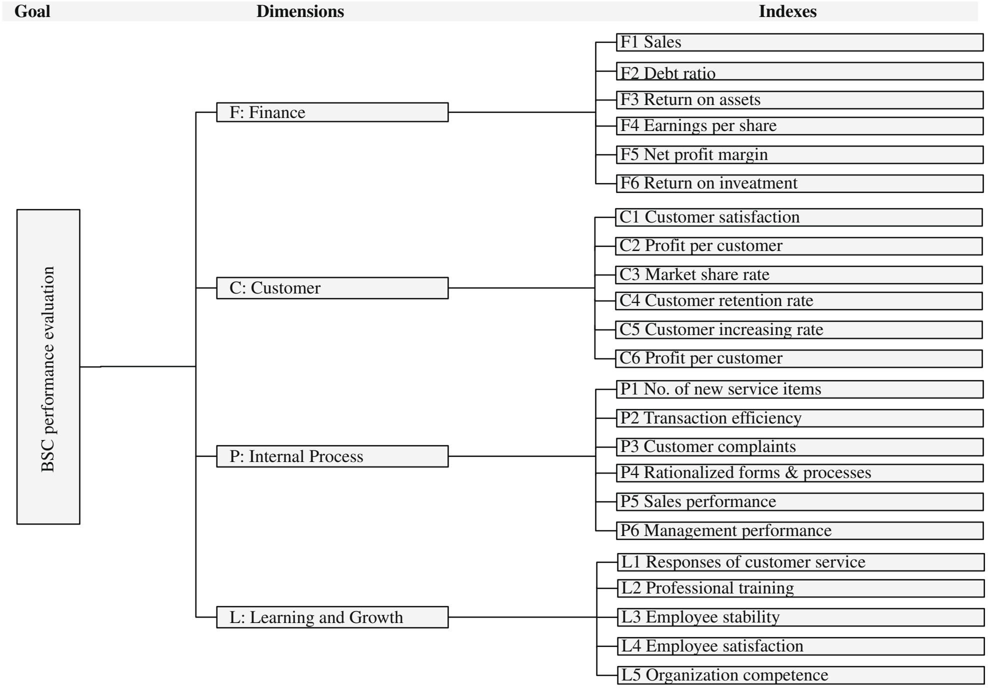

Fig. 5. Hierarchical framework of BSC performance evaluation criteria for banking.

4.2. Weights of the evaluation criteria

Based on the hierarchical framework of the BSC performance evaluation indexes, the FAHP questionnaire using FTN was distributed among the experts for soliciting their professional opinions. The relative importance (fuzzy weight) of each performance index analyzed by FAHP is listed in Table 2 using Eqs. (10) and (11). The result shows that the critical order of the four BSC dimensions for banking performance evaluation is "C: Customer (0.4101)", "F: Finance (0.3271)", "P: Internal process (0.1314)", and "L: Learning and growth (0.1314)". The top five important evaluation indexes are "C1: Customer satisfaction (0.1237)", "F3: Return on assets (0.0812)", "F4: Earnings per share (0.0784)", "C4: Customer retention rate (0.0741), " and "C2: Profit per customer (0.0715)". The least important evaluation index is "L1: Responses of customer service (0.0109)".

4.3. Ranking of the banking performance

Three banks (e.g. C Bank, S Bank, and U Bank) were taken as an illustrative example and were evaluated by the experts based on the selected evaluation criteria. Since there are differences of subjective judgments among the way experts view each evaluation criterion, the overall evaluation of the fuzzy judgment was employed to synthesize the opinions of the various experts in order to achieve a reasonable and objective evaluation. In this research, the five linguistic variables, "very dissatisfied", "not satisfied", "fair", "satisfied", and "very satisfied" were used to measure the banking performance with respect to the evaluation criteria. As shown in Fig. 3, each linguistic variable is presented by a TFN with a range of 0–100. The average fuzzy judgment values of each crite-

(19) of the average fuzzy judgment values of the three banks integrated by various experts was given by Eq. (20) as shown in Table 5. The normalized performance matrix was obtained as summarized in Table 6 by Eq. (21). According to the fuzzy weights of the BSC performance evaluation indexes by FAHP as shown in Table 2, the weighted normalized performance matrices calculated by Eq. (22) and both positive ideal and negative ideal solutions for the BSC evaluation criteria set by Eqs. (23) and (24) are shown in Table 7. Table 8 lists the separations of the ideal solution $d_i^{\oplus}$ and the negative-ideal solution d_i^{*} for the three banks by Eqs. (25) and (26). The relative closeness $RC_i^{\oplus}$ to the ideal solution and preference evaluation result derived from Eqs. (27) and (28) by TOPSIS are presented in Table 9. The relative closeness $RC_i^{\oplus}$ values for the three banks are C Bank ($RC^{*} = 0.5579$), U Bank ($RC^{*} = 0.4704$), and S Bank ($RC^{*} = 0.3521$), respectively. This implies that C Bank has the smallest gap for achieving the aspired/desired level among the three banks, whereas S Bank has the largest gap.

Similarly, VIKOR was used to rank the banking performance of the three banks based on the fuzzy weights of the BSC performance evaluation indexes by FAHP as shown in Table 2. Table 10 shows the performance matrix given by Eq. (20) with the best value $x_j^{\oplus}$ (aspired/desired levels) and the worst value x_j^{*} (tolerable/worst levels). The values of S_i and R_i computed by Eqs. (29)–(31) are shown in Table 11, while the computed value Q_i (with $v = 0, 0.5, 1$) by Eq. (32) and the preference order ranking are given in Table 12. The performance ranking order of the three banks by VIKOR is C Bank ($Q_i = 0.0000$)   U Bank ($Q_i = 0.3909$)   S Bank ($Q_i = 1.0000$). Finally, the final values and preference order ranking by these three MCDM methods, SAW, TOPSIS, and VIKOR are summarized in Table 13.

Table 5
The performance matrix $[x_{ij}]_{m\ n}$ of three banks by various experts.

Indexes	C Bank	S Bank	U Bank
F1	93.33	87.22	79.58
F2	66.67	75.00	68.75
F3	50.00	50.00	66.67
F4	41.67	58.33	75.00
F5	62.50	68.75	73.33
F6	56.25	62.50	87.23
C1	66.67	56.27	58.33
C2	72.80	41.67	37.50
C3	87.23	50.00	75.00
C4	50.00	58.33	41.67
C5	73.33	70.00	66.67
C6	75.00	62.50	87.23
P1	68.75	62.50	50.00
P2	62.50	58.33	68.75
P3	58.33	75.00	79.58
P4	75.00	66.67	75.00
P5	81.11	82.22	66.67
P6	66.67	68.75	81.11
L1	58.33	58.33	87.22
L2	81.11	73.33	75.00
L3	41.67	41.67	58.33
L4	62.50	62.50	72.78
L5	68.89	66.67	63.33

It indicates that all the ranking results are identical. However, the final values of the three banks calculated by SAW and TOPISIS are extremely close to each other. In this case, the VIKOR method is found to be a better method of assessment to clearly discriminate the banking performance.

Table 4
Fuzzy synthetic performance values of the evaluation criteria by SAW.

Criteria	C Bank	S Bank	U Bank
F	(0.29, 6.81, 112.29)	(0.32, 7.30, 120.58)	(0.39, 8.44, 132.30)
F1	(0.86, 6.71, 33.10)	(0.78, 6.15, 32.00)	(0.70, 5.45, 30.62)
F2	(0.20, 1.53, 15.20)	(0.23, 1.73, 16.42)	(0.20, 1.58, 15.50)
F3	(0.32, 3.79, 32.44)	(0.32, 3.79, 32.44)	(0.54, 5.05, 38.62)
F4	(0.24, 3.25, 26.54)	(0.42, 4.55, 33.91)	(0.62, 5.85, 39.81)
F5	(0.31, 2.42, 18.58)	(0.36, 2.66, 19.74)	(0.40, 2.90, 20.32)
F6	(0.30, 2.78, 22.01)	(0.34, 3.09, 24.29)	(0.55, 4.53, 29.36)
C	(0.71, 13.26, 169.21)	(0.49, 10.27, 146.66)	(0.54, 10.80, 148.95)
C1	(0.99, 9.22, 56.64)	(0.74, 7.78, 51.00)	(0.70, 5.45, 30.62)
C2	(0.90, 6.39, 33.21)	(0.37, 3.55, 22.99)	(0.20, 1.58, 15.50)
C3	(0.79, 5.47, 27.69)	(0.35, 2.98, 19.33)	(0.54, 5.05, 38.62)
C4	(0.38, 3.90, 28.78)	(0.50, 4.54, 31.53)	(0.62, 5.85, 39.81)
C5	(0.45, 2.73, 18.10)	(0.47, 2.43, 17.23)	(0.40, 2.90, 20.32)
C6	(0.48, 2.81, 19.79)	(0.36, 2.34, 17.59)	(0.55, 4.53, 29.36)
P	(0.09, 0.93, 19.60)	(0.08, 0.90, 19.37)	(0.09, 0.09, 18.93)
P1	(0.37, 2.67, 21.32)	(0.32, 2.43, 20.06)	(0.23, 1.95, 16.72)
P2	(0.27, 1.41, 10.01)	(0.22, 1.31, 9.91)	(0.29, 1.55, 10.98)
P3	(0.15, 0.86, 6.77)	(0.23, 1.11, 7.95)	(0.25, 1.20, 8.17)
P4	(0.26, 1.55, 12.48)	(0.22, 1.37, 11.56)	(0.26, 1.55, 12.48)
P5	(0.19, 0.80, 6.51)	(0.19, 0.80, 6.75)	(0.14, 0.64, 5.82)
P6	(0.14, 0.66, 6.18)	(0.15, 0.68, 6.31)	(0.19, 0.82, 6.93)
L	(0.09, 0.89, 17.75)	(0.08, 0.85, 17.06)	(0.10, 0.94, 18.60)
L1	(0.10, 0.51, 4.79)	(0.10, 0.51, 4.79)	(0.18, 0.80, 6.04)
L2	(0.55, 3.13, 19.94)	(0.48, 2.82, 18.70)	(0.50, 2.82, 19.23)
L3	(0.14, 0.98, 8.17)	(0.14, 0.98, 8.17)	(0.25, 1.36, 10.43)
L4	(0.26, 1.39, 11.07)	(0.26, 1.39, 11.07)	(0.33, 1.67, 12.00)
L5	(0.29, 1.63, 13.33)	(0.29, 1.63, 12.34)	(0.29, 1.38, 12.34)
Synthetic performance	(1.17, 21.89, 318.86)	(0.98, 19.32, 303.68)	(1.11, 21.06, 113.65)
BNP[a]	113.97	107.99	113.65
Ranking[b]	1	3	2

[a] BNP (Best non-fuzzy performance) = $[(U * L) + (M * L)]/3 + L$.

[b] Ranking: Rank by SAW.

Table 6
The normalized performance matrix $[r_{ij}]_{m\ \ n}$ of three banks by various experts.

Indexes	C Bank	S Bank	U Bank
F1	1.00	0.56	0.00
F2	1.00	0.00	0.75
F3	0.00	0.00	1.00
F4	0.00	0.50	1.00
F5	0.00	0.58	1.00
F6	0.00	0.20	1.00
C1	1.00	0.00	0.20
C2	1.00	0.12	0.00
C3	1.00	0.00	0.67
C4	0.50	1.00	0.00
C5	1.00	0.50	0.00
C6	0.51	0.00	1.00
P1	1.00	0.67	0.00
P2	0.40	0.00	1.00
P3	1.00	0.22	0.00
P4	1.00	0.00	1.00
P5	0.93	1.00	0.00
P6	0.00	0.14	1.00
L1	1.00	1.00	0.00
L2	1.00	0.00	0.21
L3	0.00	0.00	1.00
L4	0.00	0.00	1.00
L5	1.00	0.60	0.00

Table 7
The weighted normalized performance matrix $[v_{ij}]_{m\ \ n}$ with the ideal solutions A^* and the negative ideal solutions A^* by TOPSIS.

Indexes	C Bank	S Bank	U Bank	A^*	A^*
F1[a]	0.0604	0.0335	0.0000	0.0604	0.0000
F2[b]	0.0309	0.0000	0.0232	0.0000	0.0309
F3[a]	0.0000	0.0000	0.0812	0.0812	0.0000
F4[a]	0.0000	0.0392	0.0784	0.0784	0.0000
F5[a]	0.0000	0.0237	0.0410	0.0410	0.0000
F6[a]	0.0000	0.0107	0.0532	0.0532	0.0000
C1[a]	0.1237	0.0000	0.0246	0.1237	0.0000
C2[a]	0.0715	0.0084	0.0000	0.0715	0.0000
C3[a]	0.0527	0.0000	0.0354	0.0527	0.0000
C4[a]	0.0370	0.0741	0.0000	0.0741	0.0000
C5[a]	0.0371	0.0185	0.0000	0.0371	0.0000
C6[a]	0.0198	0.0000	0.0392	0.0392	0.0000
P1[a]	0.0438	0.0292	0.0000	0.0438	0.0000
P2[a]	0.0093	0.0000	0.0232	0.0232	0.0000
P3[b]	0.0158	0.0034	0.0000	0.0000	0.0158
P4[a]	0.0242	0.0000	0.0242	0.0242	0.0000
P5[a]	0.0113	0.0121	0.0000	0.0121	0.0000
P6[a]	0.0000	0.0019	0.0128	0.0128	0.0000
L1[b]	0.0109	0.0109	0.0000	0.0000	0.0109
L2[a]	0.0383	0.0000	0.0082	0.0383	0.0000
L3[a]	0.0000	0.0000	0.0245	0.0245	0.0000
L4[a]	0.0000	0.0000	0.0246	0.0246	0.0000
L5[a]	0.0263	0.0158	0.0000	0.0263	0.0000

A^* indicates ideal solutions and A^* indicates negative ideal solutions.

[a] Indicates the evaluation index is associated with benefit criteria and maximum is the ideal solution.

[b] Indicates the evaluation index associated with cost criteria and minimum is the ideal solution.

Table 8
The separations of the ideal solution $d_i^{\oplus}$ and the negative-ideal solution d_i^* by TOPSIS.

Banks	$d_i^{\oplus}$	d_i^*
C Bank	0.1480	0.1867
S Bank	0.1982	0.1077
U Bank	0.1731	0.1537

Table 9
The relative closeness $RC_i^{\oplus}$ to the ideal solution and preference order ranking by TOPSIS.

Banks	C Bank	S Bank	U Bank
$RC_i^{\oplus}$	0.5579	0.3521	0.4704
Ranking	1	3	2

Note: The ranking is based on the relative closeness indicating the gap to improve for achieving the aspired/desired level. The largest value means the smallest gap to achieve the ideal level.

Table 10
The performance matrix $[x_{ij}]_{m\ \ n}$ with the best value $x_j^{\oplus}$ and the worst value x_j^* by VIKOR.

Indexes	C Bank	S Bank	U Bank	$x_j^{\oplus}$	x_j^*
F1[a]	93.33	87.22	79.58	93.33	79.58
F2[b]	66.67	75.00	68.75	66.67	75.00
F3[a]	50.00	50.00	66.67	66.67	50.00
F4[a]	41.67	58.33	75.00	75.00	41.67
F5[a]	62.50	68.75	73.33	73.33	62.50
F6[a]	56.25	62.50	87.23	87.23	56.25
C1[a]	66.67	56.27	58.33	66.67	56.27
C2[a]	72.80	41.67	37.50	72.80	37.50
C3[a]	87.23	50.00	75.00	87.23	50.00
C4[a]	50.00	58.33	41.67	58.33	41.67
C5[a]	73.33	70.00	66.67	73.33	66.67
C6[a]	75.00	62.50	87.23	87.23	62.50
P1[a]	68.75	62.50	50.00	68.75	50.00
P2[a]	62.50	58.33	68.75	68.75	58.33
P3[b]	58.33	75.00	79.58	58.33	79.58
P4[a]	75.00	66.67	75.00	75.00	66.67
P5[a]	81.11	82.22	66.67	82.22	66.67
P6[a]	66.67	68.75	81.11	81.11	66.67
L1[b]	58.33	58.33	87.22	58.33	87.22
L2[a]	81.11	73.33	75.00	81.11	73.33
L3[a]	41.67	41.67	58.33	58.33	41.67
L4[a]	62.50	62.50	72.78	72.78	62.50
L5[a]	68.89	66.67	63.33	68.89	63.33

$x_j^{\oplus}$ indicates the best values for setting all the criteria functions (aspired/desired levels) and x_j^* indicates the worst values (tolerable/worst levels).

[a] Indicates the evaluation index is associated with benefit criteria and maximum is the ideal solution.

[b] Indicates the evaluation index associated with cost criteria and minimum is the ideal solution.

Table 11
The values S_i and R_i by VIKOR.

Banks	S_i	R_i
C Bank	0.0000 (1)	0.0000 (1)
S Bank	1.0000 (3)	1.0000 (3)
U Bank	0.3599 (2)	0.4219 (2)

Note: () indicates ranking order.

Table 12
The value Q_i with $v = 0, 0.5, 1$ and preference order ranking by VIKOR for sensitive analysis.

Banks	Q_i [$v = 0$]	Q_i [$v = 0.5$]	Q_i [$v = 1$]
C Bank	0.0000 (1)	0.0000 (1)	0.0000 (1)
S Bank	1.0000 (3)	1.0000 (3)	1.0000 (3)
U Bank	0.3599 (2)	0.3909 (2)	0.4219 (2)

Note: () indicates ranking order.

Table 13
Summary of final values and preference order ranking by three methods.

Banks	SAW	TOPSIS	VIKOR
C Bank	113.97 (1)	0.5579 (1)	0.0000 (1)
S Bank	107.99 (3)	0.3521 (3)	1.0000 (3)
U Bank	113.65 (2)	0.4704 (2)	0.3909 (2)

Note: () indicates ranking order.

4.4. Discussion

This research conducted a performance analysis on three banks using a FMCDM approach based on the BSC perspectives. The FAHP and the three MCDM analytical methods (i.e. SAW, TOPSIS, and VIKOR) were employed in the performance analysis for computing the fuzzy weights of the criteria, ranking the banking performance and improving the gaps of the three banks, respectively. Based on the results of the analysis, some essential findings were discussed as follows.

The FAHP adopted by the research, which combines the AHP with fuzzy set theory, can not only capture the thinking logic of human beings but also focuses on the relative importance of the evaluation criteria of the banking performance. As shown in Table 2, the result of the FAHP analysis reveals that the "Customer" dimension is the primary focus of the BSC and "customer satisfaction" is the most important evaluation index. This is because banking is a service industry, and banking performance is strongly connected to customer satisfaction. Therefore, in addition to paying attention to the financial indexes, such as ROA and EPS, which are ranked as the second and third most import indexes for sustaining a high banking performance, banks also must ensure that their customers remain loyal to them and develop new markets to attract new customers.

In addition, as mentioned in Section 3.4, the TOPSIS method is used to provide information on how to improve the gaps in each criteria so as to achieve the bank's objective (desired/aspired level) and cannot be used for ranking purpose (Opricovic & Tzeng, 2004). Therefore this research adopted the VIKOR method for ranking and improving the alternatives of this problem. Hence, based on the fuzzy weights of the evaluation criteria calculated by FAHP, the performance ranking order of the three banks using SAW is C Bank   U Bank   S Bank. The ranking order is the same as the results derived from both TOPSIS and VIKOR. However, the result found that it was evident that the final values calculated by VIKOR distinguished the banking performance among the three banks. This finding is consistent with previous studies (Chu, Shyu, Tzeng, & Khosla, 2007; Opricovic & Tzeng, 2007).

When comparing the performance of S Bank with that of the other two banks, as shown in Table 5, it is evident that S Bank has the poorest performance value in the "Customer" dimension while this is the most important BSC factor according to the experts. As far as the evaluation indexes within the "Customer" dimension are concerned, S Bank has the lowest performance value in the "market share rate" index. This implies that increasing its market share must be considered a crucial factor in that bank's growth strategy. Therefore, in addition to retaining its existing customers, S Bank should also develop new service items and/or provide more and improved promotions to attract new customers in order to keep up with the other two banks. Based on the performance analysis, it is evident that the main reason for S Bank being ranked lowest is due to the fact that its performance values from a customer perspective are poor. Therefore, for S Bank to improve its performance, it must first put more emphasis on customer satisfaction, and then on financial return.

5. Conclusions and remarks

In response to the rapid growth of service industries and the increased global competition, particularly for the banking institutions, the need for alternative controls and performance measures has attracted much attention. However, researchers are finding it difficult to measure banking performance because of the intangible nature of the products and services of the banking industry. According to the relevant literature, most studies only used financial factors to evaluate banking performance (Kosmidou et al., 2006). The present research proposed a FMCDM evaluation model for banking performance by determining a comprehensive set of evaluation criteria based on the concept of the BSC. Our proposed model embraces both financial and non-financial aspects, and optimizes the relationships of a bank. It matches the needs and requirements of the clients with the delivery processes of the bank in order to achieve the best possible customer satisfaction through effective performance.

Based on the extensive content of the BSC evaluation criteria for banking performance as selected from the relevant literature and the objective opinions synthesized from the experts, the FAHP and the three MCDM analytical methods (i.e. SAW, TOPSIS, and VIKOR) were adopted in the performance analysis for computing the fuzzy weights of the criteria, and for ranking the banking performance of three banks as an illustrative example. The relative fuzzy weights calculated by FAHP prioritize the importance of the BSC evaluation criteria for banking performance. With respect to the relative weights of the criteria, it not only reveals the ranking order of the banking performance but it also pinpoints the gaps to better achieve the bank's goal by using the MCDM analytical methods. The analysis result indicates that management should make good use of the limited resources available to improve those aspects of their business that needs improvement the most. Our proposed framework with FMCDM shows to be a feasible and effective assessment model for banking performance evaluation, and it can be applied to other institutions as well.

In conclusion, the findings of this study can be summarized as follows: 1. Integrating all the relevant experts' opinions, 23 out of the 55 evaluation indexes are selected as being suitable for banking performance in terms of BSC perspectives; 2. By applying the FAHP, the order of relative importance of the four BSC perspectives for banking performance is "C: Customer", "F: Finance", "L: Learning and growth", and "P: Internal process". The top five priorities of the evaluation indexes are "C1: Customer satisfaction", "F3: Return on assets", "F4: Earnings per share", "C4: Customer retention rate, " and "C2: Profit per customer", respectively; and 3. Using the fuzzy weights of the criteria calculated by FAHP, the ranking of the banking performance of the three banks by employing the MCDM analytical methods is U Bank, C Bank, and S Bank, respectively. Based on our findings the following suggestions are made. First, since there is no one performance evaluation index to fit all, performance evaluation indexes should be tailored to meet the organization's overall goals as well as the objectives of each individual unit. Second, the performance evaluation indexes of the BSC perspectives may not be mutually independent. Other analytical methods (e.g. fuzzy integral, Analytic Network Process, etc.) can be employed to solve the interactive and feedback relations among indexes. Third, future research may utilize several other techniques to investigate the casual relationships among performance evaluation indexes of the BSC to objectively build strategy maps. Finally, exploring more cases and conducting more empirical studies are recommended to further validate the usefulness of the proposed performance evaluation model.

Appendix. Descriptions of the selection evaluation indexes for banking performance

No.	Selection evaluation indexes	Description
1	(F1) Operating revenues	Sales revenue
2	(F2) Debt ratio	Debt divided by assets
3	(F3) Return on assets (ROA)	After-tax profit/ loss divided by average total assets
4	(F4) Earnings per share (EPS)	After-tax net earning minus preferred share dividends divided by weighted average number of shares outstanding
5	(F5) Profit margin	After-tax profit/ loss divided by total operating revenues
6	(F6) Return on investment (ROI)	After-tax profit/ loss divided by total cost
7	(C1) Customer satisfaction	Customer satisfaction of products and service
8	(C2) Profit per on-line customer	After-tax earnings divided by total number of on-line customers
9	(C3) Market share rate	Sales volumes of products and services divided by total market demands
10	(C4) Customer retention rate	Capability of keeping existing customers
11	(C5) Customer increasing rate	Growth rate of new customers
12	(C6) Profit per customer	After-tax earnings divided by total number of customers
13	(P1) No of new service items	Total numbers of new service items
14	(P2) Transaction efficiency	Average time spent on solving problems occurring during transactions
15	(P3) Customer complaint	Customer criticisms due to dissatisfaction about products and services
16	(P4) Rationalized forms and processes	Degree of procedures systemized by documentations, computer software, etc.
17	(P5) Sales performance	Successful promotion of both efficiency and effectiveness of sales
18	(P6) Management performance	Improvement of effectiveness, efficiency, and quality of each objective and routine tasks
19	(L1) Responses of customer service	Numbers of suggestions provided by customers about products and services
20	(L2) Professional training	Numbers of professional certifications or training programs per employee
21	(L3) Employee stability	Turnover of employees
22	(L4) Employee satisfaction	Employee satisfaction about both hardware and software provided by the company
23	(L5) Organization competence	Improvement of project management, organizational capability, and management by objectives (MBO), etc.

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